VOL. 23 No. 271

WIRELESS ENGINEER

APRIL 1946

Editor
W. T. COCKING, M.I.E.E.

Managing Editor
HUGH S. POCOCK, M.I.E.E.

Technical Editor
Prof. G. W. O. HOWE, D.Sc., M.I.E.E.

Editorial Advisory Board.—F. M. COLEBROOK, B.Sc., A.C.G.I. (National Physical Laboratory), L. W. HAYES, O.B.E., M.I.E.E. (British Broadcasting Corporation), Professor E. B. MOULLIN, Sc.D., M.I.E.E., A. H. MUMFORD, O.B.E., B.Sc. (Eng.), M.I.E.E. (G.P.O. Engineering Department), R. L. SMITH-ROSE, D.Sc., Ph.D., M.I.E.E. (National Physical Laboratory).

EDITORIAL

The Effective Length of a Half-wave Dipole

A HALF-WAVE dipole is usually assumed as an approximation to have a sinusoidal current distribution, and therefore to have an effective length equal to its actual length multiplied by $2/\pi$. This assumes that the field radiated by an aerial of this effective length carrying a current at every point equal to that at the centre of the half-wave dipole would be the same as that radiated by the dipole. This is true of the field strength in the equatorial plane—the direction in which we are often interested—but it is not strictly true in other directions. In the equatorial plane the radiation from all parts of the aerial arrives in phase and can be simply added ; we then have

$$\hat{\mathfrak{E}} = \hat{H} = \frac{4\pi}{10} \cdot f \frac{h}{\lambda} \cdot \frac{1}{r}$$

where h is the effective height of the earthed quarter-wave aerial or the effective length of half the dipole and r the distance in centimetres. The current i is in amperes. Putting $h = \frac{\lambda}{4} \times \frac{2}{\pi} = \lambda/2\pi$ we have $\hat{\mathfrak{E}} = \hat{H} = \frac{2fi}{10r}$.

It is interesting to note that this is exactly the same formula as that for the strength of the ordinary magnetic field H at a distance r from an infinitely long conductor carrying the same current i amperes.

As one departs from the equatorial plane, however, the equivalence of the actual dipole and the aerial λ/π long, carrying a uniform current is no longer exact. This is due to the effects of phase displacement. Fig. 1 shows the equivalent to the half-wave dipole ; its length is λ/π. In the direction shown the radiated field due to the element ds at a distance s from the centre is out of phase with that from the centre by an angle $(2\pi s/\lambda) \cos \theta$. Its effective component, *i.e.*, the component in phase with that from the centre is $\cos [(2\pi s/\lambda) \cos \theta]$, and the mean value of this from $s = -\lambda/2\pi$ to $+\lambda/2\pi$ is easily seen to be $\frac{\sin(\cos \theta)}{\cos \theta}$. Hence the strength of the field at a distance r in this direction is not simply $\frac{2i}{10r} \sin \theta$ but $\frac{2i}{10r} \sin \theta \times \frac{\sin(\cos \theta)}{\cos \theta}$. The values of $\tan \theta \,.\, \sin(\cos \theta)$ are given in Table I.

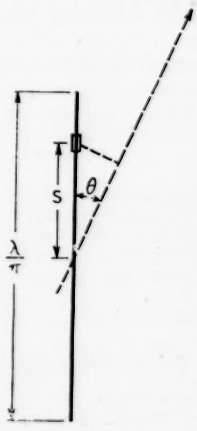


Fig. 1.

In the Editorial of April, 1945, we showed that for the sinusoidal distribution of current along the dipole the field at a distance r was given by the formula $\frac{2i}{10r} \cdot \frac{\cos[(\pi/2) \cos \theta]}{\sin \theta}$. The values for this are given in Table II and it

will be seen that as the result of the greater phase displacement, due to the longer length of aerial, the values are lower than those of the equivalent aerial. The difference is not very great, the ratio of the fields increasing from 1 per cent. for $\theta = 70°$ to about 5 or 6 per cent. for $\theta = 20°$.

It can easily be shown that as θ approaches zero, the ratio approaches $(4/\pi)$ sin 1 radian, *i.e.*, 1.071, but the values of the ratio for small values of θ are of little interest because little power is radiated in these directions. In the directions in which the greater part of the power is radiated the ratio varies from unity to about 1.03. The result of this is that, although the equivalent aerial gives the same field in the equatorial plane, its radiated power exceeds that of the half-wave dipole in the ratio of 80 to 73.2, these being the values of the radiation resistance which we obtained in the Editorial of April, 1945.

G. W. O. H.

TABLE I

θ	tan θ	Cos θ		sin (cos θ)	tan θ . sin (cos θ)
		radian	degrees		
10	0.176	0.985	56.4	0.833	0.147
20	0.364	0.94	53.8	0.807	0.294
30	0.577	0.866	49.6	0.76	0.438
40	0.839	0.766	43.8	0.69	0.579
50	1.19	0.643	36.8	0.60	0.714
60	1.73	0.5	28.6	0.48	0.831
70	2.75	0.342	19.6	0.335	0.922
80	5.67	0.174	9.96	0.173	0.981

TABLE II

θ	sin θ	$\pi/2 \cos \theta$	cos ($\pi/2$ cos θ)	$\frac{\cos (\pi/2 \cos\theta)}{\sin \theta}$
10	0.174	88.6	0.0236	0.14
20	0.342	84.6	0.0947	0.28
30	0.5	77.9	0.21	0.42
40	0.643	68.9	0.36	0.56
50	0.766	57.9	0.531	0.693
60	0.866	45.0	0.707	0.816
70	0.94	30.8	0.859	0.913
80	0.985	15.7	0.963	0.978

CARRIER-FREQUENCY AMPLIFIERS*

Transient Conditions with Frequency Modulation

By C. C. Eaglesfield

(*Mullard Radio Valve Company*)

SUMMARY.—A treatment is given of the problem of the transient response of a carrier frequency amplifier handling a signal which is frequency modulated. It is found that under certain conditions there is a simple relationship between the responses for amplitude and frequency modulation. Curves are given for staggered circuits, for various numbers of circuits and different amounts of stagger. These curves are also applicable to coupled circuits.

CONTENTS

1. Introduction.
2. General Equations for Transient Modulation Ratio.
3. Application to Staggered Circuits.
4. Numerical Results for Staggered Circuits (Frequency Modulation).

1. Introduction

THE writer has previously[2,3] dealt with the transient response of carrier frequency amplifiers to a sudden change in the amplitude of the carrier. The problem discussed here is the transient response to a sudden change in the frequency of the carrier. That is to say, the transient response of a frequency-modulation system is now considered, in place of the amplitude-modulation system previously considered.

The " amplifier " will either be a definite network of resistances, capacitances, etc., or else an unspecified algebraic function of the differential operator d/dt. It will not be defined by steady-state amplitude and phase characteristics, since it is not possible to assign to such characteristics shapes which are mutually compatible and yet simple in form.

Very little work seems to have been published on the transient response of carrier systems for either amplitude or frequency modulation, except in terms of arbitrary steady-state characteristics. Bell[1] has dealt with a single tuned circuit with frequency modulation. The present work extends the

* MS. accepted by the Editor, October 1945.

treatment of frequency modulation to any number of tuned circuits, with a reasonable flexibility of tuning adjustment.

With the extension to include frequency modulation, the writer has found it advisable to consider more closely the definitions and terms to be used. In the present work the term " Transient Modulation Ratio", usually shortened to " Modulation Ratio ", is used. This is the ratio of the modulation, however defined, at the output, to the modulation at the input, similarly defined. The definition of modulation depends on the type of system, and must in general be statistical. The necessity for a statistical definition is more apparent for frequency modulation than for amplitude modulation, and for the output than the input. While this is the idea underlying the term " Modulation Ratio", the treatment to be given is somewhat simpler. An input waveform containing an instantaneous change of amplitude or frequency is injected into the amplifier; the modulation of this simple waveform is apparent. The resulting output current is then found, and its effective amplitude or frequency evaluated, using a statistical definition. Then by comparing the variation of amplitude (or frequency) in the output with the variation in the input, the modulation ratio is obtained.

It is found that with frequency modulation, an important simplification results from restricting the modulation of the input to small values. This had previously been found to be so with amplitude modulation[3].

If also the carrier frequency coincides with the central frequency of the amplitude characteristic of the amplifier, this characteristic being symmetrical about the central frequency, there is a simple relation between the modulation ratios for the two systems, amplitude and frequency modulation. (The modulation being small for both systems).

The above statement needs qualifying: it will not be proved as such, though it may well be true. But it is found to be so for an important particular case.

The particular case referred to is that of " staggered " circuits, that is to say circuits tuned to adjacent frequencies in pairs. This arrangement has an amplitude characteristic which is symmetrical about a central frequency, and when the carrier frequency is made equal to the central frequency, then the simple relationship holds between the modulation ratios for amplitude and frequency modulation.

Since double (i.e., coupled) circuits are essentially similar to staggered circuits, the simple relationship also holds for double circuits.

In Section 2 the general equations are found for an arbitrary admittance, and the relationship is shown between the modulation ratios for amplitude and frequency modulation.

In Section 3 these equations are applied to staggered circuits.

In Section 4 numerical results are given for the modulation ratio (frequency modulation) for various numbers of staggered circuits, with different amounts of stagger. It is observed that for a given amount of stagger, the greatest slope of the modulation ratio for frequency modulation is nearly twice the greatest slope for amplitude modulation, and that subsidiary oscillations are much less. Thus if the amount of stagger is adjusted for each system so that the amount of subsidiary oscillation is roughly the same, the greatest slope for frequency modulation is probably rather more than twice that for amplitude modulation.

This comparison is for the same steady-state gain with each system.

However, it should not be forgotten in comparing the two systems, that an amplifier for frequency modulation will usually include an amplitude limiter which will sacrifice some amplification.

2. General Equations for Transient Modulation Ratio

Consider an admittance subjected to a voltage which for time t negative is sinusoidal in form with angular frequency ω_0, and for t positive has an angular frequency ω_0'. That is to say, at $t = 0$ the voltage has a sudden change of angular frequency from ω_0 to ω_0'.

The problem is to find the frequency of the current in the admittance. For t negative the current will clearly have an angular frequency ω_0, and for large positive values of t the current can be expected to have an angular frequency ω_0'.

We can thus expect a transition period during which the angular frequency of the current changes from ω_0 to ω_0'. During this transition period the form of the current will not be a pure sinusoid. It is therefore necessary to define an " effective frequency " for an arbitrary waveform. This definition must be based on statistical quantities.

The voltage waveform can be expressed operationally (in the Heaviside notation) by

$$\cos \omega_0 t + (\cos \omega_0' t - \cos \omega_0 t)\,\mathbf{1} \qquad (1)$$

since equation (1) represents a function which is equal to $\cos \omega_0 t$ for negative time and $\cos \omega_0' t$ for positive time. This follows since the Heaviside step function $\mathbf{1}$ is zero for negative time and unity for positive time. It is convenient to rewrite the expression (1) as

$$e^{j\omega_0 t}(1 - \mathbf{1}) + e^{j\omega_0' t}\,\mathbf{1} \quad .. \quad .. \qquad (2)$$

and use the real part of (2).

The admittance can be written as $A(p)$, that is, an unspecified function of the operator $p(= d/dt)$.

The current in the admittance is then

$$A(p)[e^{j\omega_0 t}(1 - \mathbf{1}) + e^{j\omega_0' t}\,\mathbf{1}]$$

Now apply the Shift Theorem to change this to :—

$$e^{j\omega_0 t}[A(j\omega_0) - A(p + j\omega_0)\,\mathbf{1}] + e^{j\omega_0' t} A(p + j\omega_0')\,\mathbf{1}$$

$$= A(j\omega_0)\left[e^{j\omega_0 t}\left\{1 - \frac{A(p + j\omega_0)}{A(j\omega_0)}\,\mathbf{1}\right\} + e^{j\omega_0' t}\frac{A(p + j\omega_0')}{A(j\omega_0)}\,\mathbf{1}\right] .. \qquad (3)$$

The factor $A(j\omega_0)$, which is independent of time, is conveniently omitted.

Write $$\frac{A(p + j\omega_0)}{A(j\omega_0)}\,\mathbf{1} = P(t) + jQ(t) \qquad (4a)$$

$$\frac{A(p + j\omega_0')}{A(j\omega_0)}\,\mathbf{1} = P_1(t) + jQ_1(t) \quad .. \quad .. \qquad (4b)$$

It will be convenient later to restrict the discussion to cases where ω_0' is very nearly equal to ω_0, so that $P_1(t)$ may be assumed equal to $P(t)$ and $Q_1(t)$ to $Q(t)$; also to cases where $Q(t)$ is zero.

Write $P(t) = P$ etc.

Expression (3) then becomes :—

$$e^{j\omega_0 t}(1 - P - jQ) + e^{j\omega_0' t}(P_1 + jQ_1)$$

The real part of this is the current in the admittance :—

$$(1 - P)\cos \omega_0 t + Q \sin \omega_0 t + P_1 \cos \omega_0' t - Q_1 \sin \omega_0' t \quad .. \quad .. \qquad (5)$$

We now require the " effective frequency " of expression (5). The effective angular frequency of a current I will be defined as :—

$$\frac{\text{Average of } dI/dt}{\text{Average of } I}$$

the averages being root-mean-square. In applying this to expression (5) it will be assumed that a duration of time can be chosen for taking the averages, which is long compared to $1/\omega_0$ and $1/\omega_0'$, but during which P etc. are sensibly constant. This amounts to neglecting the derivatives of P etc.

The effective angular frequency ω of expression (5) is given by :—

$$\omega^2 = \frac{[(1 - P)^2 + Q^2]\,\omega_0^2 + (P_1^2 + Q_1^2)\,\omega_0'^2}{(1 - P)^2 + Q^2 + P_1^2 + Q_1^2} \quad .. \quad .. \qquad (6)$$

If in the equation (4a) p is put equal to zero, then for t large :—

$$P + jQ = 1$$

i.e. $$P = 1 \text{ and } Q = 0$$

Substituting these values of P and Q in equation (6) makes $\omega = \omega_0'$ for t large. For t negative, $P = Q = P_1 = Q_1 = 0$. Thus $\omega = \omega_0$ for t negative.

Equation (6) thus describes a transition in ω from ω_0 to ω_0'.

Now assume that ω_0' is nearly equal to ω_0. Equation (6) becomes :—

$$\omega = \omega_0 + \frac{P_1^2 + Q_1^2}{(1 - P)^2 + Q^2 + P_1^2 + Q_1^2}(\omega_0' - \omega_0)$$

The modulation ratio for frequency modulation is thus

$$\frac{P_1^2 + Q_1^2}{(1 - P)^2 + Q^2 + P_1^2 + Q_1^2}$$

But if ω_0' is nearly equal to ω_0, P_1 will be nearly equal to P, and Q_1 to Q. Assuming $P_1 = P$ and $Q_1 = Q$, the modulation ratio is

$$\frac{P^2 + Q^2}{(1 - P)^2 + P^2 + 2Q^2} \quad .. \quad .. \qquad (7)$$

If further, $Q = 0$, the ratio is

$$\frac{P^2}{(1 - P)^2 + P^2} \quad .. \quad .. \quad .. \qquad (8)$$

The expressions (7) and (8) are the small modulation expressions.

The frequency transition has now been expressed in terms of the functions $P(t)$ etc. The problem of amplitude modulation can also be expressed in terms of these functions.

We now consider the same admittance subjected to a voltage which at $t = 0$ undergoes a sudden change of amplitude,

being sinusoidal before and after the change with the same angular frequency ω_0.

If the amplitude is 1 before the change and $(1 + m)$ after the change, the voltage can be written as

$$e^{j\omega_0 t}(1 + m\,\mathbf{1}) \quad .. \quad .. \quad .. \quad (9)$$

the real part to be taken.

The current is then :—

$$A(p)e^{j\omega_0 t}(1 + m\,\mathbf{1})$$
$$= e^{j\omega_0 t}\left[A(j\omega_0) + mA(p + j\omega_0)\,\mathbf{1}\right]$$
$$= A(j\omega_0)\,e^{j\omega_0 t}\left[1 + m\,\frac{A(p + j\omega_0)}{A(j\omega_0)}\,\mathbf{1}\right]$$

and by equation 4(*a*), this can be written :—

$$A(j\omega_0)e^{j\omega_0 t}\left[1 + mP + jmQ\right] \quad .. \quad (10)$$

The factor $A(j\omega_0)$, which is independent of time, is conveniently omitted. Doing this and taking the real part of (10), we get for the current

$$(1 + mP)\cos\omega_0 t - mQ\sin\omega_0 t \;..\; (11)$$

To obtain the amplitude of (11), it is necessary first to define an effective amplitude for an arbitrary waveform. Such a definition must be statistical. The effective amplitude of a current I will be defined as

$$(\text{Average of } I)$$

the average being root-mean-square.

In applying this to (11), it will be assumed that a duration of time can be chosen, long compared to $1/\omega_0$ but during which P and Q are sensibly constant.

The effective amplitude of expression (11) is :—

$$\sqrt{(1 + mP)^2 + (mQ)^2} \quad .. \quad .. \quad (12)$$

The expression (12) can be written

$$1 + m\left[\frac{\sqrt{(1 + mP)^2 + (mQ)^2}}{m} - \frac{1}{m}\right]$$

so that the modulation ratio is :—

$$\frac{\sqrt{(1 + mP)^2 + (mQ)^2}}{m} - \frac{1}{m} \quad .. \quad (13)$$

The modulation ratio is thus in general dependent on m ; it becomes independent of m if m is either very large or very small :—

$$\sqrt{P^2 + Q^2}\ (\text{for } m \text{ very large}) \quad .. \quad (14a)$$
$$P \quad (\text{for } m \text{ very small}) \quad .. \quad (14b)$$

If $Q = 0$, it is P in both cases.

Comparing the modulation ratio for frequency and amplitude modulation, it is clear that while both are generally expressible in terms of P and Q, the one is given by the other only when $Q = 0$.

We then have, equations (8) and (14),

$$\text{Modulation Ratio} = \frac{P^2}{(1-P)^2 + P^2}\ \text{(F.M.)}$$

where P is the modulation ratio (A.M.).

Thus if the modulation ratio (A.M.) is known numerically, it is a simple matter to obtain the modulation ratio (F.M.).

For this simple relation to hold, it is necessary that the function Q be zero, the depth of frequency modulation be very small, and the depth of amplitude modulation be either very small or very large.

In the next section these results will be applied to the particular case of staggered circuits.

3. Application to Staggered Circuits

The modulation ratio for amplitude modulation has already been given [3] for the case of an amplifier chain with "staggered circuits." The reader is referred to the original reference for the full derivation, but for the sake of completeness a brief summary is included here.

Consider an amplifier chain consisting of n stages of which Fig. 1 is a typical stage.

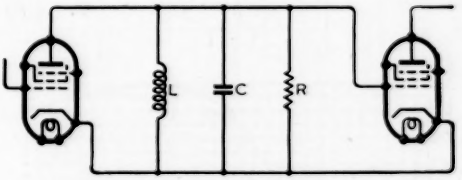


Fig. 1. *A typical amplifier stage.*

For each stage C and R are the same, but in alternate stages L has one of two values, so that one set of circuits is resonant at one frequency and the other set, equal in number, at a second frequency. n is an even integer. Let the two resonant angular frequencies be $(\omega_0 + \omega_1)$ and $(\omega_0 + \omega_2)$ where ω_0 is the injected angular frequency as in Section 2.

Write

$$a = 1/2CR$$
$$k = \frac{\omega_1 - \omega_2}{2a}$$
$$h = \frac{\omega_1 + \omega_2}{2a}$$
$$z = kat$$
$$N = n/2$$
$$\nu = N - \tfrac{1}{2}$$

k is the "stagger" coefficient, and h the "detune" coefficient.

By letting ω_0 tend to infinity, while ω_1 and ω_2 are kept constant, the expressions for the functions P and Q are obtained in the following form (see equation (15) of reference[3]) :—

$$\begin{matrix} P \\ Q \end{matrix} = A \int_0^z e^{N-z/k} \begin{matrix} \cos \\ \sin \end{matrix} \left(\frac{h}{k} z - N\theta \right) \left(\frac{z}{N} \right)^{\nu} J_{\nu}(z)\, dz \qquad .. \quad .. \quad (15)$$

In this equation $J_{\nu}(z)$ is the Bessel function of the first kind of order ν and argument z, and A and θ are given by

$$A = \left[\frac{(1 + k^2 - h^2)^2 + 4h^2}{4k^4} \right]^{N/2} \frac{\sqrt{2\pi N}\, N^N\, e^{-N}}{N\,!} \qquad .. \quad .. \quad (16)$$

$$\tan \theta = \frac{2h}{1 + k^2 - h^2} \qquad .. \quad .. \quad (17)$$

The problem of finding the modulation ratio with amplitude and frequency modulation for staggered circuits is thus formally solved.

Now consider the case of $h = 0$. This makes $\theta = 0$, and $Q = 0$. Equation (15) becomes

$$P = A \int_0^z e^{N-z/k} \left(\frac{z}{N} \right)^{\nu} J_{\nu}(z)\, dz \qquad .. \quad (18)$$

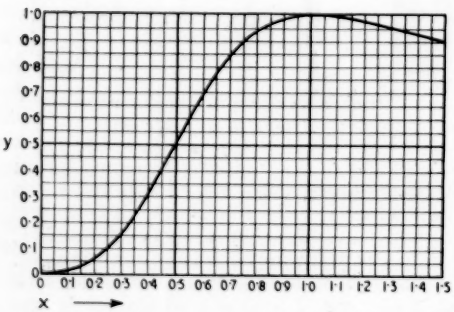


Fig. 2. *The function* $y = \dfrac{x^2}{(1-x)^2 + x^2}$

Since $Q = 0$, the modulation ratio for amplitude modulation (small modulation) is given by P ; and for frequency modulation (small modulation) by

$$\frac{P^2}{(1-P)^2 + P^2}$$

The condition $h = 0$ implies that the "detune" ratio is zero : that is, that the carrier frequency coincides with the central frequency of the amplitude-frequency characteristic of the amplifier chain.

4. Numerical Results for Staggered Circuits

A number of numerical results have already been given[2,3] for the modulation ratio (A.M.) of staggered circuits. To convert these results into the modulation ratio (F.M.), we require the function.

$$y = \frac{x^2}{(1-x)^2 + x^2} \qquad .. \quad .. \quad (19)$$

x being the modulation ratio (A.M.), and y the modulation ratio (F.M.). A curve of this function is shown in Fig. 2 (for numerical convenience a table was constructed) ;

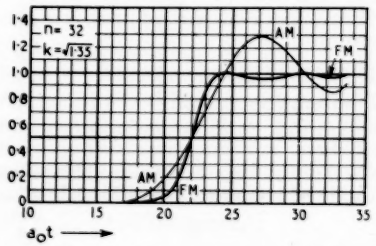


Fig. 3. *The modulation ratio for amplitude and frequency modulation for a particular case of staggered circuits.*

y always lies between 0 and 1 ; $dy/dx = 2$ for $x = 1/2$; and y droops rather slowly from 1 for x greater than 1.

Fig. 3 shows both the modulation ratio (A.M.) and the modulation ratio (F.M.) for a particular case ($n = 32$, $k = 1.35$). It will be seen that although the modulation ratio (A.M.) is very oscillatory, the restrictive action of (19) makes the modulation ratio (F.M.) much less so, and the maximum slope is greater.

If the modulation ratio (A.M.) had had its greatest slope when its value was 1/2, the greatest slope (F.M.) would have been twice the greatest slope (A.M.). In all the examples given, the greatest slope (F.M.) is somewhat less than twice the greatest slope (A.M.).

Fig. 4 shows the modulation ratio (F.M.) for a number of values of n and k. On the curves the greatest slope and the stationary values are marked. Also with each curve a sketch, which is not to scale, of the corresponding amplitude-frequency characteristic

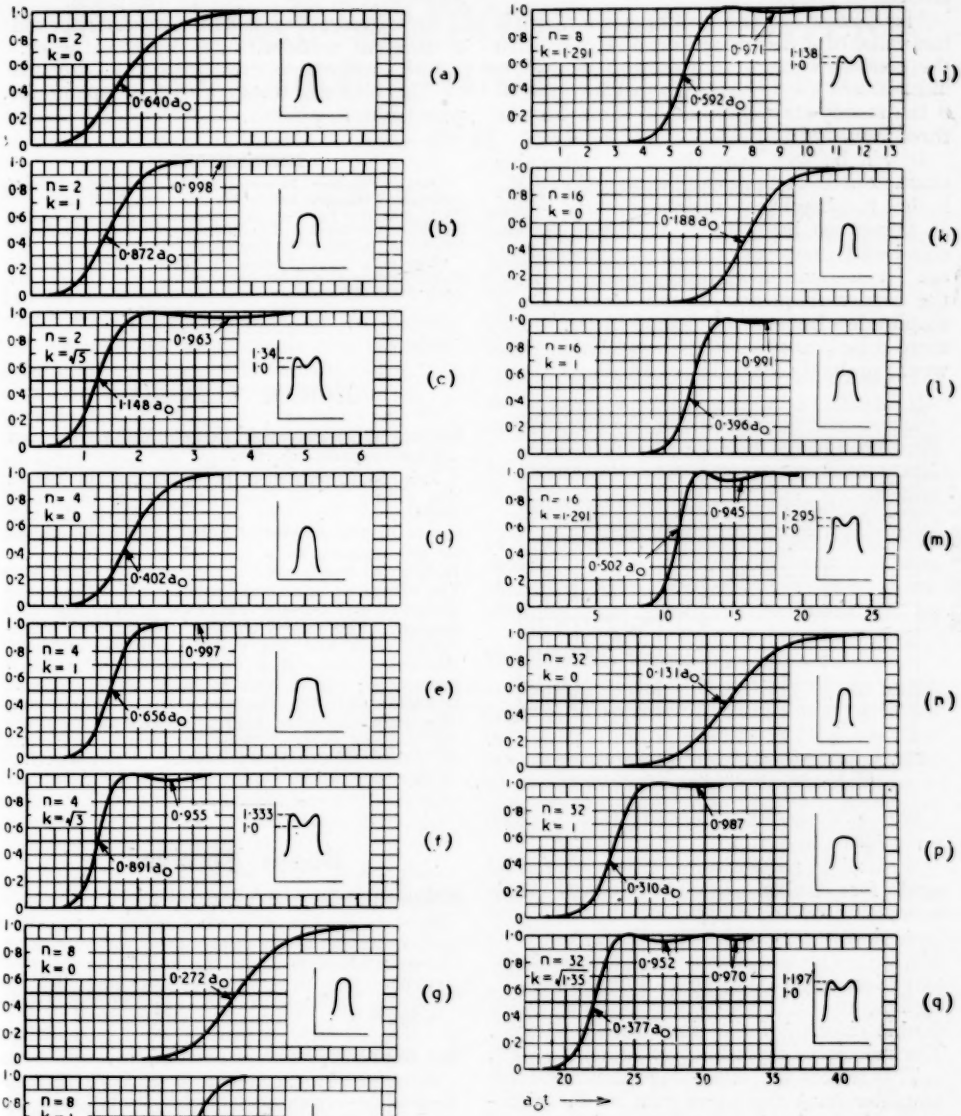


Fig. 4. *The modulation ratio (F.M.) for various numbers of staggered circuits and various amounts of stagger.*

is given. This is useful for visualizing the amount of stagger.

The time scale used in Fig. 4 is $a_0 t$. a_0 is the value of a corresponding to $k = 0$, with the condition that the steady-state gain is independent of k. Thus for each value of n the steady-state gain is the same for the three values of k.

It will be seen that the most noticeable characteristic of the modulation ratio (F.M.) is that it " bounces " against the final value. It cannot go beyond it. It should be remembered, however, that the curves given are for a small modulation depth and for the carrier frequency at the centre of the amplitude - frequency characteristic. For more complicated cases the modulation ratio would probably look rather different.

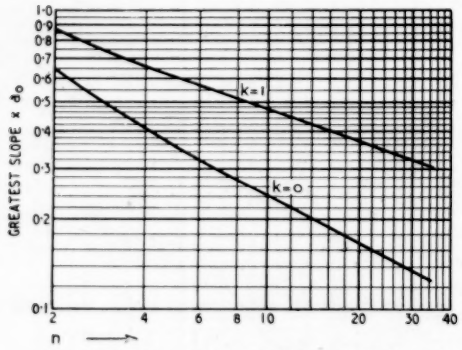


Fig. 5. *The greatest slope of the modulation ratio (F.M.) for various numbers of circuits.*

Fig. 5 shows the greatest slope against n for $k = 0$ and 1.

With slight modification, the curves can be used for double circuits (perhaps more commonly called coupled circuits). These have been dealt with previously[2]. Thus Fig. 4(*l*), which is drawn for a sixteen-stage amplifier with staggered single circuits, applies to an eight-stage amplifier with double circuits, if the time scale is altered. The rule is to replace a_0 by $2a_0k\sqrt{(1 + k^2)}$.

This makes the eight-stage double-circuit amplifier have the same gain as the eight-stage single-circuit amplifier. The method however breaks down for $k = 0$.

Acknowledgements

The writer is conscious of a debt to Mr. D. P. Dalzell for some general discussion of the matter in this paper as well as the two previous papers on the transient state of carrier frequency amplifiers, in which acknowledgement on specific points was made.

He is grateful to several assistants for aid at different times with the numerical work.

And finally he is grateful to the Directors of The Mullard Radio Valve Company for permission to publish.

REFERENCES

[1] Bell, "Transient Response in Frequency Modulation," *Philosophical Magazine*, Vol. 35, No. 242, p. 143, March 1944.

[2] Eaglesfield, "The Unit Step Response of Carrier Frequency Amplifiers with Single and Double Circuits," *Wireless Engineer*, Vol. XXII, No. 266, p. 523, November 1945.

[3] Eaglesfield, "Carrier Frequency Amplifiers, Transient Response with De-tuned Carrier," *Wireless Engineer*, Vol. XXIII, No. 270, p. 67, March 1946.

BOOK REVIEW

Systems of Electric and Magnetic Units. By Jean Flambouriaris, pp. 120.

This book, published in Athens, is in Greek. The author is the Chief Engineer of the Greek Ministry of Transport and he is obviously well versed in the subject of units. He acknowledges his indebtedness to la Revue Générale and to the Bulletin of the Société Française des Electriciens. The subject of units and dimensions and the choice of fundamental units are dealt with in the first chapter. The second chapter is devoted to mechanical units and the various systems employed. The final chapter deals with electric and magnetic units, the c.g.s. systems, the Giorgi system, rationalization, the Gaussian and Lorentz systems. The book concludes with a comprehensive index and a large table showing the dimensions of all the units in the various systems. G.W.O.H.

Books Received

Television Today and Tomorrow

By Lee de Forest, Ph.D., D.Sc., D.Eng. A popular account of the history of television with an elementary technical description of present day methods. Pp. 176. Hutchinson's Scientific & Technical Publications Ltd., 47, Princes Gate, London, S.W.7. Price 16s.

The Decibel Notation

By Vepa V. Lakshmana Rao, B.E. (Elect.). A detailed explanation of the decibel and the phon and their applications. Conversion tables are included. Pp. 179 + xvi. Addison & Co. Ltd., Madras, India. Price 13s. 6d.

Atomic Spectra

By R. C. Johnson, M.A., D.Sc. This book covers the theory of atomic spectra including the effects of electric and magnetic fields external to the atom. Pp. 120 + viii. Methuen & Co. Ltd., 36, Essex Street, Strand, London, W.C.2. Price 5s.

GRAPHICAL SYMBOLS FOR FILTERS AND CORRECTING NETWORKS*

By G. H. Foot

Introduction

THE present British Standard Symbols† for Filters and Correcting Networks are unsatisfactory because they are limited to a few simple types. Moreover, there is no rational manner in which they can be modified to illustrate new developments or special facilities. As they are also not particularly easy to draw or to understand, for they can be confused with the British Standard Symbols for alternating currents, and as they fail to give much information which is desirable, it is suggested that they should be replaced by the comprehensive system to be described.

The Valve Symbol

It will be useful to consider the reasons for the success of the conventional method of representing a thermionic valve. Four lines are used to show the three electrodes of a triode and its envelope. The nature and function of each electrode is suggested by the type of line, and the circle surrounding them indicates the boundary of the evacuated space. The external connections are shown by taking short lines to this boundary, and these can be numbered to correspond with a standard valve base. The valve characteristics may be specified sufficiently by giving the values of the principal valve parameters. More often the code of the valve is quoted, and this information must be obtained from the maker's catalogue.

Such a symbol has proved to be very satisfactory. In a simple way it gives a great deal of information, and this is presented so that it can be understood by technologists without difficulty. Also, and this is very important, the method of modifying the symbol for more complicated and special types of valves, is obvious. The interpretation of the modified symbols is usually manifest and ambiguity is improbable. The symbolization of screen-grid and pentode valves, and valves with metallization, indirectly-heated cathodes, gas-filling, and variable-mu properties has not been difficult. Many other variants have been treated successfully, including types which are really several valves in one with internal connections and screening. By modifying the shape of the envelope, the symbol for a cathode-ray tube was devised.

All the valve symbols discussed have been found to be satisfactory for the pencil sketches of engineers, yet they are scarcely altered when carefully drawn. The attempts of draughtsmen to make improvements in detail, and the divergences resulting from the independent progress of different countries, have not caused any confusion.

The Rectifier Symbol

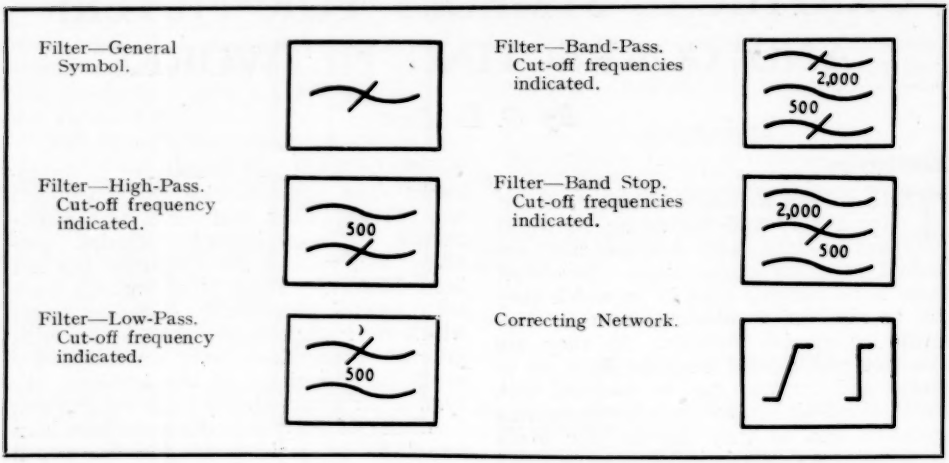
The solution of the problem of the valve symbol is apparently simple and obvious, but in it can be discovered the cardinal principle to be followed in successful electrical symbology. It is necessary that a symbol should enable the apparatus it represents to be identified, and its essential characteristics to be ascertained, without the need for memorizing an arbitrary code. This can be done by basing it on the physical structure of the apparatus. The symbol becomes an elementary picture of the equipment. Sometimes this is not convenient because nothing characteristic of the apparatus is denoted by its construction, or because different arrangements are used to develop the same property.

An example is the rectifier where the symbol diagrammatically represents its performance. When the rectifier is of the multiple type however, the symbol is repeated to indicate the relative positions of the plates. The combination of the electrical and mechanical properties thus portrayed, has been found to present the information which the engineer must have, without including anything unessential.

* MS. accepted by the Editor, November 1945.

† British Standard Graphical Symbols for Telephony, Telegraphy, and Radio Communication. No. 530; 1937.

BRITISH STANDARD SYMBOLS FOR FILTERS AND CORRECTING NETWORKS



The Basis of the Proposed New Symbols

The shape, size, and mechanical details of a filter (or correcting network) do not determine its electrical behaviour. The value of the filter is assessed after examining its attenuation-frequency characteristic, which is usually given graphically. To an engineer nothing is more strongly associated with a filter than this graph, and nothing more quickly conveys the essential information about performance than a glance at it. It is on this graph that it is suggested the filter symbol should be based. Additional particulars will be necessary, for example, details of the external connections and a description of the construction as far as it affects these. The symbol should therefore include this information.

The present British Standard Symbols and the proposed new symbols are shown in the Tables together with some examples to clarify their use. The advantages of the new symbols are most evident in the case of the more complex filters, such as that of Example 4. Filters of this kind are in common use, but the existing symbols are quite inadequate to give the information which is generally required.

Correcting Networks and Amplifiers

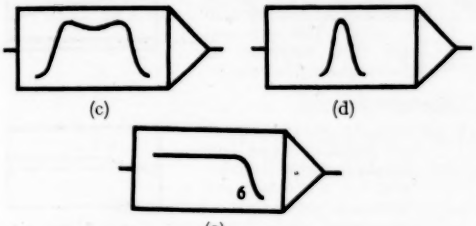
The symbol for a correcting network is based on the attenuation-frequency characteristic of the network. This avoids the considerable limitation of having only one symbol for every network, irrespective of the type of correction it provides. Thus networks with more attenuation at high frequencies than at low frequencies are represented by (a) and if the attenuation is less at high frequencies than at low frequencies then the symbol becomes (b)

(a) (b)

It will be observed that a correcting network can always be distinguished, because the line representing the attenuation goes from side to side of the rectangle. In a filter symbol this line rests on the bottom of the rectangle.

It is usual to obtain the response of radio receiving apparatus as the voltage gain at various frequencies for a constant output. This had led to the custom amongst radio engineers of plotting the gain-frequency characteristic, and this will be a mirror image of the attenuation-frequency characteristic. Such apparatus should, however, be regarded as amplifiers with various types of response curves. They could conveniently be symbolized in the manner shown by

sketches (c), (d), and (e), which will be self-explanatory.



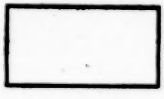
It is not our purpose at the moment to be concerned with this type of symbol, except to show how it could be included in the scheme. It will be sufficiently obvious that the combination of the rectangle with the usual triangular sign removes any possible ambiguity, as well as providing a uniform and valuable method of representing amplifiers comparable with the method for representing filters already discussed.

Many other variations might be described, but not advantageously at the moment. They might easily confuse the reader as to the basis of the proposed plan. Indeed as it is claimed that the new symbols can be developed logically as required, no elaboration is necessary.

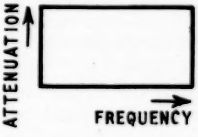
SUGGESTED NEW SYMBOLS FOR FILTERS

Derivation

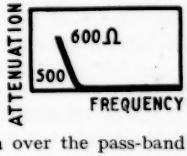
The phyiscal boundary of the filter (or correcting network) is indicated by a rectangle.



The rectangle is considered as a sheet of graph paper with the lower edge as frequency axis, and the left-hand edge as an attenuation axis, the origin being the lower left-hand corner.



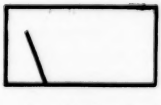
The general form of the attenuation - frequency characteristic is drawn and cut-off frequencies in kc/s are inserted in the acute angle. The characteristic impedance is written over the pass-band.



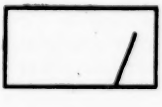
The thickened line in the pass-band shown in the derivation is unnecessary and merges into the boundary of the symbols.

New Symbols

High-Pass.

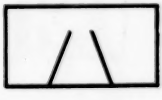


Low-Pass.



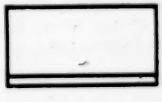
Band-Pass.

Band-Stop.

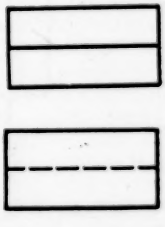


A band-pass filter having two pass-bands which do not overlap.

Unbalance in the filter is shown by a line representing the earth line.

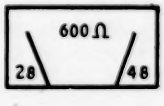


Balance in a filter is shown by a line in the centre—solid for a physical earth connection, otherwise dotted.



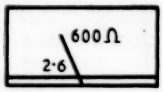
Example 1

A band-pass filter with a pass-band between 28 kc/s and 48 kc/s and with a characteristic impedance of 600 Ω.



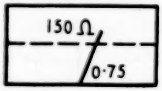
Example 2

An unbalanced high-pass filter with a cut-off frequency of 2.6 kc/s and a characteristic impedance of 600 Ω.



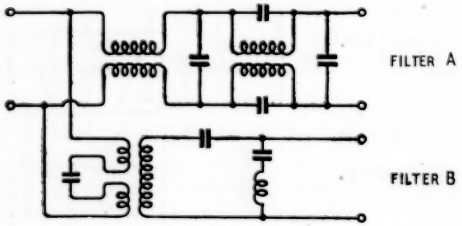
Example 3

A balanced low-pass filter (no physical earth connection) with a cut-off frequency of 750 c/s, and a characteristic impedance of 150 Ω.



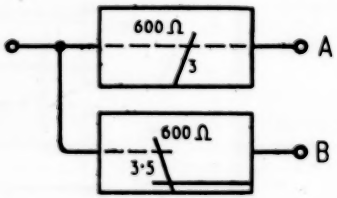
Example 4

The following circuit is of a pair of filters used for separating a voice-frequency telephone circuit from carrier circuits operating on the same cable pair.



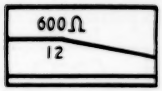
Filter A is a balanced low-pass filter with a cut-off frequency of 3 kc/s, a characteristic impedance of 600 Ω, and no physical earth connection. Filter B is a high-pass filter with a cut-off frequency of 3.5 kc/s and a characteristic impedance of 600 Ω. It has a balanced input where it is connected to the low-pass filter, but one of the coils is used as a transformer and an unbalanced output is provided.

These filters are represented in the new system as follows :—



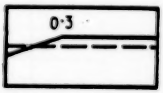
Example 5

A 600 Ω unbalanced correcting network has a constant attenuation to 12 kc/s. The attenuation then decreases as the frequency is raised.



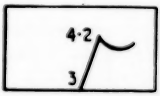
Example 6

A correcting network of the balanced type (no physical earth connection) has a constant attenuation above 300 c/s, but below this frequency the attenuation decreases as the frequency is lowered.



Example 7

A low-pass filter has a cut-off frequency at 3 kc/s, and a point of infinite (i.e. high) attenuation at 4.2 kc/s.



Summary.—As the British Standard Symbols for Filters and Correcting Networks are inadequate, new symbols are proposed. It is shown how the scheme can be extended to include amplifiers.

The advantages of the new symbols are :

(1) They are based on performance graphs which display information which an engineer requires, and which he associates closely with the apparatus itself.

(2) The understanding of the symbols does not depend on memorizing an arbitrary code.

(3) They give information often required, but for which no provision is made in the British Standard Symbols.

(4) The elaboration of a symbol to illustrate a new development or a special feature, is a natural and logical process. An error in the interpretation of a modified symbol is improbable.

(5) The new symbols are very simple, and confusion with other symbols is impossible.

(6) The proposed scheme is flexible, and not likely to be inadequate for future requirements.

SOME EXPERIMENTS WITH LINEAR AERIALS*

By J. S. McPetrie, D.Sc., Ph.D., M.I.E.E., and J. A. Saxton, Ph.D., B.Sc., A.M.I.E.E.

(Communication from The National Physical Laboratory)

SUMMARY.—The first part of this paper describes a method by which the polar diagram of any rectilinear aerial can be determined. The method is based on the assumption that elementary current waves are induced in the aerial by an incident electromagnetic field, the final current in the aerial being the summation at each point of the effect of these current waves after reflection at each end of the aerial. If the lengths of the aerial on either side of the point of connection of the aerial to the receiver are equal or differ by an integral number of half wavelengths, the analysis is considerably simplified as the correct polar diagram is obtained if reflection is assumed to occur only once at each end of the aerial. Figures are given in the paper showing close agreement between the theoretical and experimental polar diagrams obtained for aerials of various lengths between one-half and five half-wavelengths. It is also shown experimentally that the polar diagram of any aerial is the same when that aerial is used for transmission or reception.

Part II describes an experimental investigation which has been made of the front-to-back signal ratio of a receiving aerial system consisting of a half-wavelength dipole and single parasitic aerial. When the latter acted as a reflector its optimum length and spacing from the main receiving aerial were approximately 0.5 and 0.1 of the wavelength respectively. For this condition the pick-up in the backward direction was 11 decibels below that in the forward direction. When the parasitic aerial acted as a director, however, the optimum back to front signal ratio was approximately 20 decibels and the length of parasite and spacing from the receiving aerial 0.47 and 0.05 of the wavelength. The overall gain of this aerial system over a simple half-wavelength dipole was also highest when the parasitic aerial was adjusted roughly as a director for optimum back-to-front signal ratio.

PART I

1. Introduction

IT has been shown[1] that the current distribution in an aerial may be quite different depending upon whether the aerial is used for transmission or reception. This result suggests the comparison of other properties of transmitting and receiving aerials. A method is described which is suitable for the determination of the polar diagram of a linear receiving aerial. At each point along the aerial two elementary current waves are assumed to be induced by an incident electromagnetic wave. One current wave travels directly towards the point at which the receiver is connected to the aerial and the other wave arrives at the same point after reflection at one end of the aerial. If the difference in the lengths of the aerial on the two sides of the receiver connection is zero or an integral number of half-wavelengths, the correct polar diagram of the aerial is obtained by integrating with proper phase relation the total currents at the receiver connection due to the direct elementary current waves from each point and the indirect waves, the latter after reflection at that end of the aerial at which reflection first takes place. The present paper is limited to a consideration of the cases in which this condition applies.

* MS. accepted by the Editor, October 1945.

2. Description of Method

The polar diagram of a receiving aerial represents the variation of input voltage to the receiver with aerial orientation. As the input impedance of the receiver is unaltered by aerial rotation this voltage is proportional to the current in the receiving aerial at the point in the latter from which the aerial output is fed to the receiver. The polar diagram of a receiving aerial, therefore, can be determined completely if the aerial current at this point is known for all orientations of the aerial.

Let AB in Fig. 1 represent a rectilinear aerial and O the point in it at which the aerial output is applied to the receiver either directly or by some coupling device such as a transmission line. If a plane electromagnetic wave of amplitude E is incident on the aerial in the direction making an angle θ with its length, the voltage induced in an

element of length δx at P is $E \sin\theta \,.\, \delta x$. This voltage developed across the element at P gives rise to two current waves in AB, one travelling in the direction PA and the other in the opposite direction PB. These two elementary current waves undergo successive reflections at each end of the aerial but, in the transient state before these reflections take place, the paths PA and PB appear at P

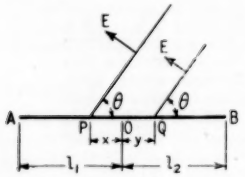


Fig. 1. *Diagrammatic representation of linear aerial.*

as two infinitely long lines each having impedance Z_0 where $2Z_0$ is the input impedance of an infinitely long aerial in free space of the same material and having the same cross-section as the aerial AB.* The equivalent circuit of the aerial measured at P in the transient state can, therefore, be represented by the circuits shown in Figs. 2(*a*) and (*b*). The initial transient current, as seen from Fig. 2(*b*), has amplitude $\frac{E \sin\theta \,.\, \delta x}{2Z_0}$. This represents the amplitude of each of the two waves emanating from P.

The final current in the aerial at any point can be obtained by adding the effects of the elementary waves originating at each point along the aerial and their subsequent reflections at each end of the aerial, and any discontinuity at the receiver connection. For the present purpose in which the magitude of the current at O is not required, but only its variation with θ, the orientation of the aerial with respect to the incident wave, it is more convenient to consider the two portions OA and OB of the aerial separately. The current at O, Fig. 1, due to OA is constituted by the vectorial addition at O of all the elementary current waves induced in OA travelling towards O, and all the indirect waves after reflection at the end A. If the magnitude of the current at O due to OA is desired, the subsequent reflections of these current waves at each end of the aerial must be taken into account

For the usual conditions on ultra-short waves, in which the lengths OA and OB are equal, or differ by an integral number of half-wavelengths, the effect of these subsequent reflections at A, B and O can be neglected when only the variation of aerial current with aerial orientation is required. For the determination of the polar diagram of the portion OA of the aerial, therefore, only the current waves induced in OA travelling directly to O and the indirect waves after a single reflection at A will be taken into account in the present paper. The reason that the latter indirect waves must be considered is because the inducing field gives rise at each point to current waves travelling in opposite directions along the aerial, and the phases along OA of the current waves travelling originally in the direction OA are related to incident radiation just as much as the current waves in the opposite direction AO. If the same procedure is followed for the section OB and the currents so obtained for OA and OB added vectorially a formula

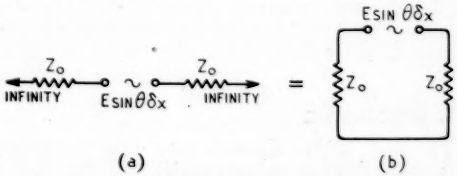


Fig. 2. *Equivalent circuit in transient state of aerial having characteristic impedance* Z_0.

results which, with the limitation as to the position of O in AB mentioned above, gives the correct variation of the current at O with rotation of the aerial and, therefore, the polar diagram of the aerial AB when used for reception. It is shown in an appendix that for an aerial having impedance $2Z_0$, as defined above, the current at a point l_1 and l_2 from each end of the aerial such that

$$l_1 - l_2 = n\,\lambda/2$$

and derived in the above manner from the summation of elementary current waves is given by

$$I = A - jB \quad .. \quad .. \quad ., \quad (1)$$

in which A and B are, respectively

$$A = \frac{E\lambda}{2\pi Z_0 \sin\theta}\left\{\sin\left(\frac{4\pi l_1}{\lambda}\cos^2\frac{\theta}{2}\right) + \sin\left(\frac{4\pi l_2}{\lambda}\sin^2\frac{\theta}{2}\right) - \sin\frac{4\pi l_1}{\lambda}\right\} \quad (2)$$

* If the aerial AB were equivalent to a transmission line having uniformly distributed constants along its length, $2Z_0$ would represent the characteristic impedance of the aerial. It would also be the input impedance of an infinite length of this transmission line. Schelkunoff[2], however, has shown that for an infinitely long aerial in free space the characteristic and input impedances are not equal.

$$B = \frac{E\lambda}{2\pi Z_0 \sin\theta}\left\{1 - \cos\left(\frac{4\pi l_1}{\lambda}\cos^2\frac{\theta}{2}\right) - \cos\left(\frac{4\pi l_2}{\lambda}\sin^2\frac{\theta}{2}\right) + \cos\frac{4\pi l_1}{\lambda}\right\} \quad .. \quad .. \quad (3)$$

When the receiver is connected to the centre of the aerial the current I (equation 1) is given by

$$I = \frac{E\lambda}{\pi Z_0 \sin\theta}\left\{\cos\left(\frac{2\pi l}{\lambda}\cos\theta\right) - \cos\frac{2\pi l}{\lambda}\right\} \quad .. \quad .. \quad (4)$$

where l is half the length of the aerial.

3. Experimental Procedure

Two methods were used to determine experimentally the polar diagrams of various aerials and thus to test the validity of the formulae given above. One method was followed when only a commercial type of short-wave receiver was available and the second when a field-strength measuring set could be utilized. In each case the transmitting and receiving aerials were arranged horizontally at equal heights above ground of about 1 m and connected respectively to a small oscillator and receiver by means of twisted flexible leads about 2 m in length.

For measurements with the commercial receiver the transmitter was modulated and the audio voltage developed across the telephone terminals of the receiver was observed. This output voltage was maintained constant as the receiving aerial was rotated about a vertical axis by suitably adjusting a piston type of attenuator inserted in the lead between the modulated oscillator and the radiating aerial. In this way the field-strength at the receiver could be varied over a range of 40–50 decibels. As, in this method, the output and, therefore, the input voltage to the receiver is fixed the relation between these two voltages is not required to be known. The necessary adjustment of the attenuator at the transmitter was made by an observer acting under instruction from a second observer at the receiver. This procedure was relatively simple as the distance between transmitting and receiving aerials was only about 50 m.

When a field-strength measuring set was available, however, the oscillator was unmodulated and observation made of the variation of input voltage to the receiver with receiver aerial orientation, no adjustment of transmitter output being required. Because in this latter method, all measurements were made at the receiver it was more convenient than the first method and was followed whenever possible. Various wavelengths between 50 and 100 cm were used in the experiments, but those described in this report were all made on a wavelength 100 cm.

4. Comparison between Theoretical and Experimental Results

Figs. 3 to 8 give at (*a*) the polar diagrams derived from the above analysis for the length of aerial shown in each illustration, while corresponding diagrams obtained experimentally are given at (*b*). The agreement between theory and experiment is seen to be very good. It is particularly interesting to note from Figs. 6 and 7 that in the case of an aerial two wavelengths long the large change in polar diagram with change in

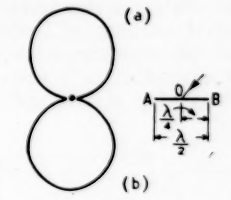


Fig. 3.

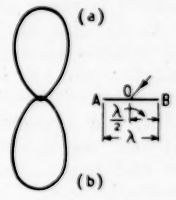


Fig. 4.

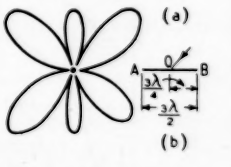


Fig. 5.

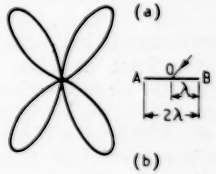


Fig. 6.

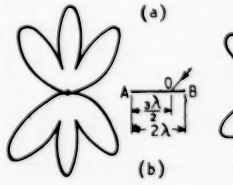


Fig. 7.

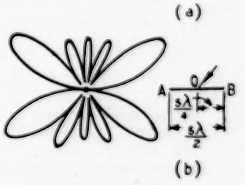


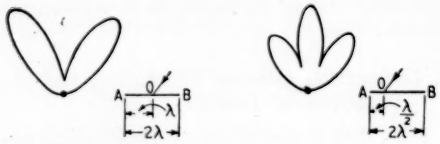
Fig. 8.

Figs. 3, 4, 5, 6, 7 and 8. (*a*) *Theoretical and* (*b*) *experimental polar diagrams of receiving aerials of different lengths. AB represents the aerial and O the point of connection to receiver.*

change in polar diagram with change in

position of the connection to the receiver is completely accounted for theoretically.

The analysis outlined in the report, therefore, appears satisfactory for the determination of the polar diagram of any receiving aerial in which the connection to the receiver is made at the centre of the aerial or at a point an integral number of half-wavelengths nearer one end than the other. The analysis has been extended to a consideration of rhombic receiving aerials and, again, the agreement between the theoretical and experimental results is good. The polar diagrams of the aerials shown in Figs. 6 and 7 when used as transmitting aerials are given in Figs. 9 and 10. A comparison of Fig. 6 with Fig. 9, and Fig. 7 with Fig. 10 shows that the polar diagram of an aerial is the same whether it is used for transmission or reception. In the determination of the polar diagram of an aerial, therefore, it is largely a matter of personal preference whether the aerial is assumed to be associated with a transmitter or a receiver.



Figs. 9 and 10. *Experimental polar diagrams of transmitting aerials. AB represents the aerial, and O the point of connection to transmitter.*

Conclusions

The analysis given in the paper is based on the assumption that a receiving aerial of given length is equivalent to an equal length of a uniform transmission line and also that the wavelength of the current waves induced in the wire by external radiation is the same as the wavelength in free space of that radiation. The latter condition is only satisfied if the velocity of propagation of the waves along the aerial is the same as that of light. The close agreement between experiment and theory arrived at on these assumptions shows that such conditions can be considered as exact in most practical cases. Two further conclusions which may be deduced from the work described in this paper are that the polar diagram of any aerial is the same for transmission and reception, and that the correct polar diagram of an aerial can be determined on the assumption that the voltage developed in the aerial is due to discrete current waves induced in the aerial by an incident electromagnetic field.

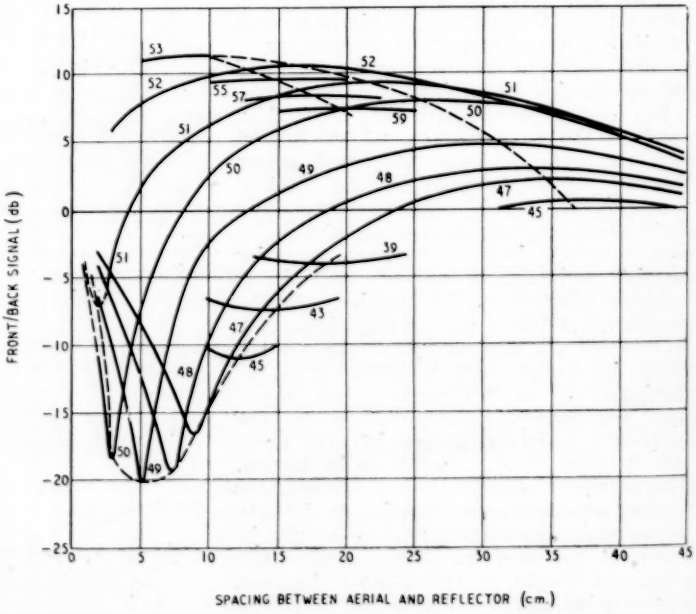


Fig. 11. *Front/back signal with various reflector lengths. λ/2 dipole aerial. λ=1.05 metres. The numbers on the curves represent reflector lengths.*

PART II

1. Introduction

An experimental investigation has been made on certain features of a simple aerial system consisting of a half-wavelength aerial and parasitic reflector. The main purpose of the experiments was to determine the front-to-back signal ratio of such an aerial system and the factors on which this ratio depends.

2. Experimental Procedure and Results

A horizontal electric field with a wavelength of 1.05 m was set up at a receiving site by means of a small screened oscillator connected by a length of about 2 m of twin flexible leads to a horizontal dipole located at a distance of about 50 m from the receiver. The aerial system under investigation was erected at the receiver site, the half-wavelength aerial being placed horizontally and connected to the input terminals of a field-strength measuring set by a vertical length of about 1 m of twin flexible leads. The parasitic aerial was supported at its centre parallel to the main receiving aerial and was constructed of telescopic brass tubing so that its length could be varied. The distance between the main and parasitic aerials could also be varied and the complete receiving aerial system was rotatable about a vertical axis. For various distances between the main and parasitic aerials and for different lengths of the latter, polar diagrams of the receiving aerial system were determined.

Complete diagrams were taken only for a limited number of conditions as the information most required was simply the front-to-back signal ratio. For this purpose the receiver input voltage with the parasitic aerial at a given distance on the side of the main aerial remote from the transmitting aerial was compared with that obtained when the receiving aerial system was rotated about a vertical axis through an angle of 180°.

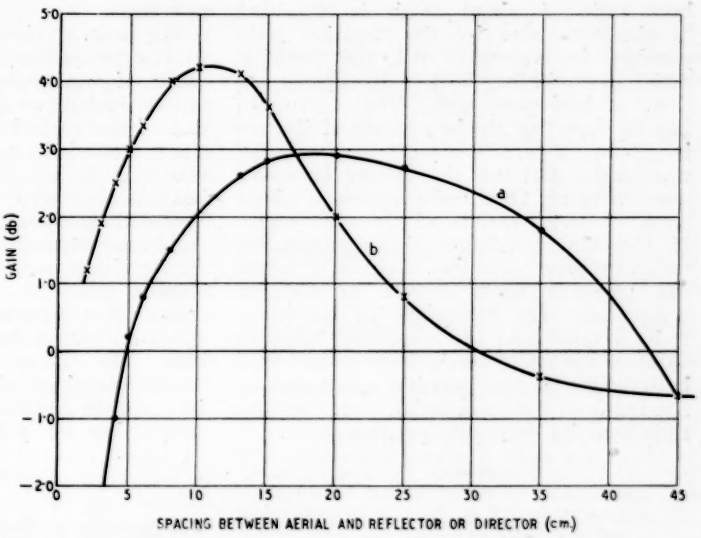


Fig. 12. *Gain of λ/2 dipole aerial with reflector (a) and director (b) over that of λ/2 dipole without reflector.*

The results are shown graphically in Fig. 11, in which the ordinates give this ratio of input voltages, termed front-to-back ratio and expressed in decibels, for various spacings between the main and parasitic aerials and for a series of lengths of the latter. The dotted curves in this figure give approximately the optimum front-to-back ratio for any distance.

The front-to-back signal ratio, measured in the direction of the transmitter, can be either positive or negative. When it is positive, the parasitic aerial is on the further side of the receiving aerial, and is known as a reflector. When the ratio is negative the parasitic aerial is on the side of the receiving aerial nearer the transmitter and is known as a director.

It will be seen from Fig. 11 that the maximum positive front-to-back ratio is obtained with a reflector 53 cm in length, which is very nearly equal to half the wavelength (105 cm) used in the experiments;

also the appropriate distance for this maximum condition is 10 cm, or one-tenth of the wavelength. Near the maximum positive front-to-back ratio the magnitude of this ratio does not appear to be critically dependent either on the spacing between the main and parasitic aerials or on the length of the latter. The maximum positive front-to-back ratio of about 11 db is exceeded in absolute value by the negative ratio possible, for example, with the parasitic aerial 49 cm in length, that is, slightly less than a half-wavelength. The curves in Fig. 11 show that the best length of director is 49 cm, or approximately 0.47 of the wavelength, and that the spacing for maximum negative front-to-back ratio is about 5 cm, or one-twentieth of the wavelength. If, therefore, maximum discrimination between two directions 180° apart is required, the parasitic aerial should be used as a director. The magnitude of the front-to-back ratio near the maximum negative value of the latter varies more rapidly with both length of the parasitic aerial and its spacing from the main aerial, than the same ratio near its maximum positive value.

The curves in Fig. 12 show the gain over a single half-wavelength aerial of the aerial system with parasitic aerial adjusted in length and spacing approximately for the maximum positive and negative front-to-back ratios, respectively. The gain with the parasitic aerial acting as a director is seen to be greater than when it is used as a reflector.

The main receiving aerial was connected to the field-strength measuring set by a length of twin flexible leads having a characteristic impedance of about 100 ohms, one end of the leads being inserted directly at the centre of the aerial and no attempt at matching made. The condition for matching was satisfied roughly with the distance between main and parasitic aerials in the region corresponding to maximum positive front-to-back signal ratio. Measurements showed that the matching was far from correct for the small spacing required for optimum negative front-to-back signal ratio. No change in the curves in Fig. 11 would have been obtained if matching had been made between receiving aerial and feeder, but for the small spacings, an in-

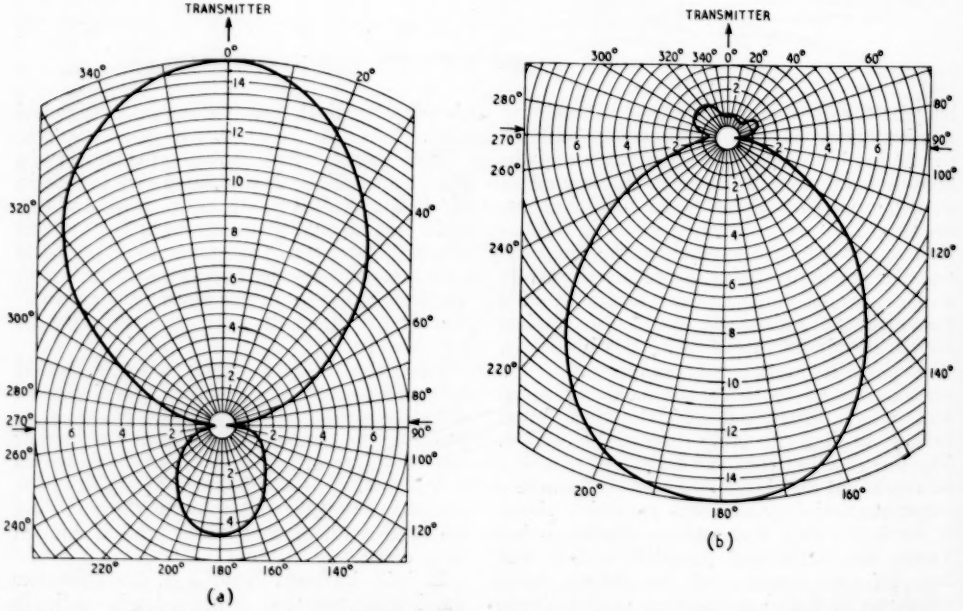


Fig. 13. *Polar diagrams for a dipole with reflector (a) and director (b), at a wavelength of 105 cm. The reflector length was 53 cm spaced 18 cm from the dipole and the gain over the simple dipole was 2.9 db. The director had a length of 49 cm spaced 5 cm from the aerial and had a gain of 2.2 db over a simple dipole.*

crease in gain would have resulted over the curves given in Fig. 12. This increase in gain for such spacing would have made the maximum gain of the aerial system with the parasite acting as a director, greater than that given in Fig. 12 but little, if any, change would have occurred for the greater spacing for optimum reflector conditions. With the parasitic aerial under optimum conditions for a director, therefore, both a better absolute value of front-to-back signal ratio and gain over a half-wavelength dipole would be obtained over those possible with the parasitic aerial acting as a reflector.

In Figs. 13 (*a*) and (*b*) typical polar diagrams are given for the two cases in which the parasitic aerial acts as a reflector and director respectively. The lengths of parasitic aerial correspond to maximum positive and negative front-to-back signal ratios, but the optimum spacing from the main aerial is correct only for the parasitic aerial acting as a director. As mentioned above, however, the change with distance in either polar diagram or front-to-back signal ratio is not critically dependent on spacing near the maximum positive value of that ratio. Incidentally, although the two curves in Figs. 13 (*a*) and (*b*) show an improvement in front-to-back signal ratio for the smaller spacing between main and parasitic aerials, the gains in the two cases over a simple half-wavelength at the receiver are approximately the same.

Acknowledgements

The work described above was conducted as part of the programme of the Radio Research Board to whom confidential reports were circulated during 1941. This paper is published by permission of the Department of Scientific and Industrial Research.

APPENDIX

As shown in Part I of the paper, two elementary current waves each having magnitude $\frac{E \sin\theta \,.\, \delta x}{2 Z_0}$ are induced in the element δx at P (Fig. 1) by the field E incident on the aerial in the direction making an angle θ with its length. These two current waves travel in opposite directions from P. It is convenient to refer all phases to that of the elementary current wave I_0 induced by the field E at the point O in the aerial at which the final aerial current is required.

If λ be the wavelength of the incident radiation the current wave at P lags in phase on that of I_0 by the angle $\frac{2\pi x}{\lambda}\cos\theta$ due to the phase of the field at P lagging on the phase of E at O by the same amount. In travelling directly to O there is an additional retardation in phase of $\frac{2\pi x}{\lambda}$, provided that the wavelength of the current wave in the aerial is the same as that of the incident radiation in free space. The current wave induced at P which travels directly to O lags at O in phase on I_0, therefore, by the angle

$$\frac{2\pi x}{\lambda}(1+\cos\theta) = \frac{4\pi x}{\lambda}\cos^2\theta/2.$$

The component of the total current at O due to all the elementary current waves in OA which travel directly towards O is

$$\frac{E\sin\theta}{2Z_0}\int_0^{l_1} exp\left(-j\frac{4\pi x}{\lambda}\cos^2\theta/2\right)dx = j\frac{E\lambda\tan\theta/2}{4\pi Z_0}\left\{exp\left(-j\frac{4\pi l_1}{\lambda}\cos^2\theta/2\right)-1\right\} \quad .. \quad .. \quad (1)$$

The initial current wave at P which travels in the direction PA also has amplitude $\frac{E\sin\theta\,\delta x}{2Z_0}$ and lags in phase on I_0 by the angle $\frac{2\pi x}{\lambda}\cos\theta$. In travelling from P to A and thence to O along the aerial there is an additional phase lag of $\frac{2\pi}{\lambda}(2l_1 - x)$. The reflection coefficient at the end A of the aerial must be such that there is no current at this point. This condition is satisfied if there is π change in phase of the current wave on reflection at A. The current wave from P, therefore, which arrives at O after reflection at A, lags at O in phase on I_0 by the angle

$$\frac{2\pi}{\lambda}(x\cos\theta + 2l_1 - x) + \pi = \frac{4\pi}{\lambda}(l_1 - x\sin^2\theta/2) + \pi$$

Hence, the component of the total current arriving at O from OA after reflection at A is

$$-\frac{E\sin\theta}{2Z_0}\int_0^{l_1} exp\left\{-j\frac{4\pi}{\lambda}\left(l_1 - x\sin^2\theta/2\right)\right\}dx = j\frac{E\lambda\cot\theta/2}{4\pi Z_0}\left\{exp\left(-j\frac{4\pi l_1}{\lambda}\cos^2\theta/2\right) - exp\left(-j\frac{4\pi l_1}{\lambda}\right)\right\} \quad .. \quad .. \quad (2)$$

For a point Q in OB (Fig. 1) distant y from O the elementary current wave leads that at O in phase by the angle $\frac{2\pi y}{\lambda}\cos\theta$. The total phase lag on I_0 of the current wave arriving at O directly from Q is, therefore,

$$\frac{2\pi y}{\lambda}(1-\cos\theta) = \frac{4\pi y}{\lambda}\sin^2\theta/2$$

The total current at O from OB due to all such elementary waves from OB is

$$\frac{E\sin\theta}{2Z_0}\int_0^{l_2} exp\left(=j\frac{4\pi y}{\lambda}\sin^2\theta/2\right)dy = j\frac{E\lambda\cot\theta/2}{4\pi Z_0}\left\{exp\left(-j\frac{4\pi l_2}{\lambda}\sin^2\theta/2\right)-1\right\} \quad .. \quad .. \quad (3)$$

Remembering the π phase change on reflection at B the phase at O of the current wave from Q travelling towards B lags on that of I_0 by the angle

$$\frac{2\pi}{\lambda}\left\{2l_2 - y(1+\cos\theta)\right\} + \pi = \frac{4\pi}{\lambda}(l_2 - y\cos^2\theta/2) + \pi$$

Therefore, the component at O of the indirect current wave from OB is

$$-\frac{E\sin\theta}{2Z_0}\int_0^{l_2} exp\left\{-j\frac{4\pi}{\lambda}\left(l_2 - y\cos^2\theta/2\right)\right\}dy$$

$$= j\frac{E\lambda\tan\theta/2}{4\pi Z_0}\left\{exp\left(-j\frac{4\pi l_2}{\lambda}\sin^2\theta/2\right)\right.$$

$$\left. - exp\left(-j\frac{4\pi l_2}{\lambda}\right)\right\} \quad .. \quad .. \quad (4)$$

The total current at O from AB due to the direct and indirect current wave, the latter after one reflection, is

$$R = A - jB$$

where, remembering that $l_1 - l_2 = \frac{n\lambda}{2}$, we have :—

$$A = \frac{E\lambda}{4\pi Z_0}(\tan\theta/2 + \cot\theta/2)\left\{\sin\left(\frac{4\pi l_1}{\lambda}\cos^2\theta/2\right)\right.$$

$$\left. + \sin\left(\frac{4\pi l_2}{\lambda}\sin^2\theta/2\right) - \sin\frac{4\pi l_1}{\lambda}\right\}$$

$$= \frac{E\lambda}{2\pi Z_0\sin\theta}\left\{\sin\left(\frac{4\pi l_1}{\lambda}\cos^2\theta/2\right)\right.$$

$$\left. + \sin\left(\frac{4\pi l_2}{\lambda}\sin^2\theta/2\right) - \sin\frac{4\pi l_1}{\lambda}\right\}$$

Similarly

$$B = \frac{E\lambda}{2\pi Z_0\sin\theta}\left\{1 - \cos\left(\frac{4\pi l_1}{\lambda}\cos^2\theta/2\right)\right.$$

$$\left. - \cos\left(\frac{4\pi l_2}{\lambda}\sin^2\theta/2\right) + \cos\frac{4\pi l_1}{\lambda}\right\}$$

When $l_1 = l_2 = l$ it is found that

$$|R| = (A^2 + B^2)^{\frac{1}{2}}$$

$$= \frac{E\lambda}{\pi Z_0\sin\theta}\left\{\cos\left(\frac{2\pi l}{\lambda}\cos\theta\right) - \cos\frac{2\pi l}{\lambda}\right\}$$

$$.. \quad .. \quad (5)$$

Three special cases of equation (5) are interesting

(*a*) $l = (2n+1)\lambda/4$

$$|R| = \frac{E\lambda}{\pi Z_0\sin\theta}\left\{\cos\left[\frac{2n+1}{2}\pi\cos\theta\right]\right\} \quad (6)$$

(*b*) $l = (2n+1)\lambda/2$

$$|R| = \frac{2E\lambda}{\pi Z_0\sin\theta}\left\{\cos^2\left[\frac{2n+1}{2}\pi\cos\theta\right]\right\} \quad (7)$$

(*c*) $l = n\lambda$

$$|R| = \frac{2E\lambda}{\pi Z_0\sin\theta}\sin^2(n\pi\cos\theta) \quad .. \quad .. \quad (8)$$

REFERENCES

[1] N. von Korshenewsky : "Electric Oscillations." *Zeit. f. Tech. Phys.*, 1929, Vol. 10, pp. 604–608.

[2] S. A. Schelkunoff : "Theory of Antennas of Arbitrary Size and Shape." *Proc. I.R.E*, 1941, Vol. 29, p. 493.

CORRESPONDENCE

Letters of technical interest are always welcome. In publishing such communications the Editors do not necessarily endorse any technical or general statements which they may contain.

Pulse Modulation

To the Editor, "Wireless Engineer."

SIR,—I wish to challenge some of the statements made by Roberts and Simmonds in Section 1.4 of their article in the November issue of *Wireless Engineer* (Vol. 22, p. 543).

Equation (1) is the Fourier series corresponding to a waveform which repeats itself at intervals equal to $2\pi/\omega$ (I shall, except where noted to the contrary, follow the notation used in Roberts and Simmonds' article). The quantity a_ν is a constant (Sect. 1.4.1, line 9) determined by the shape of the pulses and the integer ν, and for a chain of rectangular pulses of amplitude A it is given by

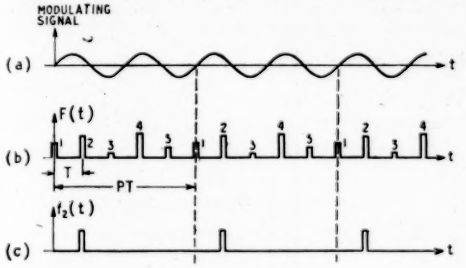
$$a_\nu = 2AS\frac{\sin\nu\pi S}{\nu\pi S}$$

Since A has been assumed constant, the Fourier series corresponding to the amplitude-modulated pulse chain cannot be obtained by replacing it by $A(1 + m\cos pt)$, which is variable. Furthermore, the modulated pulse chain does not repeat itself at intervals of $2\pi/\omega$, so that at least two of the conditions under which equation (1) was derived no longer hold. A similar criticism can be applied to the procedure followed in Sections 1.4.2 and 1.4.3.

A chain of rectangular pulses is a succession of discrete phenomena to which the idea of continuous variation cannot be applied. For example, in the case of delay-modulation the shape of the pulse does not alter. In the case of amplitude and width-modulation the shape of the pulse is determined by the modulating signal at some instant before the pulse occurs. In all three types of modulation the property of the pulse chain which is being varied is determined by the modulating signal at a number of instants equally spaced along the time axis; what happens to the modulating signal between these instants has no effect on the modulated pulse chain. The mathematical method of obtaining the Fourier series of the modulated chain of pulses must therefore be one which will take this into account. Such a method may be outlined in the following way.

Suppose that the ratio of the pulse to the modulating frequencies, ω/p, is a rational fraction $r = P/Q$, where P and Q are integers prime to one another. The waveform of the modulated pulse chain then repeats itself at intervals equal in length to P pulses or Q cycles of the modulating frequency,

since these are equal by the definition of P and Q. In order to fix our ideas let us consider a pulse chain amplitude-modulated by the sinusoidal modulating signal shown in Fig. 1 (*a*). For a ratio $r = 5/2$, the resultant pulse chain will be as shown in Fig. 1 (*b*). This (infinite) pulse chain may be divided into P pulse chains of period PT ($T = 2\pi/\omega$), which I will refer to as *partial pulse chains*; let the Fourier series of the nth one be denoted by $f_n(t)$. Such a partial pulse chain, viz. $f_2(t)$ is shown in Fig. 1 (*c*).



If $F(t)$ denote the Fourier series of the complete modulated pulse chain we have

$$F(t) = \sum_{n=1}^{P} f_n(t).$$

If we assume that the epoch of the leading edge of the pulse is

$$t_n = (n - 1)T - \tfrac{1}{2}ST$$

and that the amplitude of the nth partial pulse chain is given by

$$A_n = A_0(1 + m \sin pt_n)$$

then we obtain for the amplitude of the component in the modulating frequency in $F(t)$ the expression

$$mA_0S \frac{\sin \pi S/r}{\pi S/r}$$

and that the amplitude of the component with pulsatance $\nu\omega \pm p$ is

$$mA_0S \frac{\sin [\pi(\nu \pm 1/r)S]}{\pi(\nu \pm 1/r)S}$$

The differences between these results and those derived from equation (3) are almost trivial, nevertheless in the case of width- and delay-modulated pulse chains the results of a similar chain of reasoning are very different. For a width-modulated pulse chain the amplitude of the component in the modulating frequency is

$$mAS \frac{J_1(2\pi mS/r)}{\pi mS/r} \longrightarrow mAS$$

as $\pi mS/r \to 0$. $J_1(z)$ is the Bessel function of the first kind of order unity. This departure from linearity is 1.1 per cent for $\pi mS/r = 0.15$, and might not be noticed in practice. For delay-modulated pulse chains a component in the modulating frequency is present and its amplitude is

$$\frac{AS\phi_{max}}{r} \frac{\sin \pi S/r}{\pi S/r} \frac{J_1(\phi_{max}/r)}{\phi_{max}/2r} \longrightarrow \frac{AS\phi_{max}}{r}$$

as $r \to \infty$, that is for a given pulse chain it is proportional to the frequency of the modulating signal.

This latter result is a contradiction of the statement by Roberts and Simmonds in Section 1.4.3 that such a component does not exist. The fact that the amplitude of the signal varies linearly with frequency in no way makes the system private, because it is only necessary to apply it to an RC integrating circuit to obtain a signal whose amplitude is independent of the frequency, provided $1/pCR \ll 1$ for the lowest modulating frequency required. Of course the amplitude of this signal would be small, both because S would be chosen small in a phase-modulated system and also because $1/r < 1$.

I hope to publish a fuller account of these calculations in the near future.

RONALD B. SHEPHERD.
University College of Swansea.

Rationalization of Publication

To the Editor, "The Wireless Engineer."

SIR,—I would like to make a few comments about Mr. Bell's letter regarding Rationalization of Publication. I am, in general, in complete agreement with his remarks, but I would like to point out that there are many books on limited branches of science or engineering which have been published in this country, many of them before books on the same subjects appeared in America.

I am rather surprised at his suggestion that the types of publication mentioned in his letter are not available as British books by British authors and I certainly consider he is unduly pessimistic in this respect.

I quite agree that the better type of book is international in scope and I consider that technical books should give references to the origin of papers and books from which information has been taken, without regard to the nationality of the authors. In this respect I find that the majority of American books and publications signally fail and I must also admit that many British books fail in the same way but the failing is nothing like so great as in the case of American publications. That I have always held this view may be gauged by the fact that in my book "Time Bases" the following numbers of references are made.

British references	..	..	..	58
American ,,	..	..	..	29
German ,,	..	..	..	5
French ,,	..	..	..	3
Italian ,,	..	..	..	1

I am, however, also in agreement with Mr. Bell's suggestion that a definite policy regarding the rationalization of publication is urgently needed and I would like to suggest that the three main Engineering Institutions of this country, together with perhaps the Physical Society, the Chemical Society and others of like nature, might combine for the purpose of originating a common policy in connection with these publications.

It would, of course, be necessary and advisable to obtain the advice of such bodies as the National Physical Laboratory, the Royal Society and various book and periodical publishing firms.

O. S. PUCKLE.
Amersham, Bucks.

NOISE FACTOR OF VALVE AMPLIFIERS

By N. R. Campbell, Sc.D., F.Inst.P.,

V. J. Francis, B.Sc., F.Inst.P., A.M.I.E.E., and E. G. James, Ph.D., B.Sc.

(Communication from the Research Staff of The M.O. Valve Company at the G.E.C. Research Laboratories, Wembley, England)

(Concluded from page 83 of the March issue)

10. Definition of and Formulae for Noise Factor

We are now in a position to discuss noise factor. If the signal generator is independent (in the sense of the last paragraph of Section 2) of the other sources of noise, then the noise of the practical receiver can be written $v_s^2 + v_p^2$ where v_s^2 is the noise at the output due to the signal generator and v_p^2 that due to the other sources. In the ideal receiver v_p^2 is zero by hypothesis; further the signal voltage is the same for the practical and the ideal receiver. We have therefore, from the fundamental definition of N in Section 1,

$$N = \frac{v_s^2 + v_p^2}{v_s^2} \quad .. \quad .. \quad .. \quad (10.1)$$

$$= 1 + \frac{v_p^2}{v_s^2} \quad .. \quad .. \quad .. \quad (10.2)$$

We now consider an amplifier of n stages; then

$$v_s^2 = 4k'T \int_0^\infty R_s |A_1|^2 . |A_2|^2 \ldots df \quad .. \quad .. \quad (10.3)$$

and v_p^2 will be the sum of the components from all stages which from (9.3) is given by

$$\begin{aligned} v_p^2 = {} & 4k'T \int_0^\infty Z_1 |A_2|^2 |A_3|^2 \ldots |A_n|^2 \, df \\ & + 4k'T \int_0^\infty Z_2 |A_3|^2 |A_4|^2 \ldots |A_n|^2 \, df \\ & + \ldots \\ & + 4k'T \int_0^\infty Z_{n-1} |A_n|^2 \, df \\ & + 4k'T \int_0^\infty Z_n \, df \end{aligned} \quad .. \quad .. \quad (10.4)$$

It should be noted that Z_1 by definition of v_p^2 does not include the noise from the generator.

Accordingly

$$N = 1 + \frac{\left(\int_0^\infty Z_1 |A_2|^2 |A_3|^2 \ldots |A_n|^2 \, df + \ldots \right) \left(\ldots + \int_0^\infty Z_{n-1} |A_n|^2 \, df + \int_0^\infty Z_n \, df \right)}{\int_0^\infty R_s |A_1|^2 |A_2|^2 \ldots |A_n|^2 \, df} \quad .. \quad .. \quad (10.5)$$

If the amplifier consists of one stage only, (10.5) reduces to

$$N = 1 + \frac{\int_0^\infty Z_1 \, df}{\int_0^\infty R_s |A_1|^2 \, df}$$

$$= 1 + \frac{\int_0^\infty \sum_{pq} (G_{pq} + G_{pq0}) |\rho_{kl,\,pq}|^2 \, df}{\int_0^\infty R_s |A_1|^2 \, df} \quad .. \quad .. \quad (10.6)$$

where $\sum_{pq} G_{pq}$ does *not* include R_s.

In any practicable receiver the range of each integral can be restricted to finite limits, f_1 to f_2, outside which the integrand is zero. So long as the integrand varies appreciably within the limits (which may not be the same for all the integrals), the integrals must be retained, and no general simplification of (10.5) is possible. On the other hand, it is assumed in much noise theory that all the quantities concerned are constant when they are not zero. If this assumption is permissible the integral signs in (10.5) can be omitted, so that (10.5) becomes

$$N = 1 + \frac{1}{R_s |A_1|^2} \left\{ Z_1 + \ldots + \frac{Z_r}{|A_2|^2 \ldots |A_r|^2} + \ldots + \frac{Z_n}{|A_2|^2 \ldots |A_n|^2} \right\} \quad .. \quad .. \quad (10.7)$$

An intermediate condition is when the assumption is true of stage 1, but not of the others. Then (10.5) becomes

$$N = 1 + \frac{1}{R_s|A_1|^2}\left\{\frac{\int_0^\infty Z_1|A_2|^2 \ldots |A_n|^2\,df + \ldots + \int_0^\infty Z_r|A_{r+1}|^2 \ldots |A_n|^2\,df + \ldots}{\int_0^\infty |A_2|^2 \ldots |A_n|^2\,df}\right\} \quad (10.8)$$

11. Application of Noise Factor Formulae

The later stages of the amplifier do not include a signal generator. But the input circuits contain effective generators of E.M.F. derived from the output voltage of the previous stages. In defining the A_r's it was said that the input circuit of the rth stage may be taken to be one having an admittance equal to the admittance looking into the output terminals of the $(r-1)$th stage, which is ${}_{r-1}Y_{kli}$, and containing a generator whose E.M.F. is the voltage across those output terminals when they are disconnected from the input terminals of the rth stage. In calculating the ρ's for the rth stage, i.e. ${}_r\rho_{kl,pq}$, the admittance ${}_{r-1}Y_{kli}$ must be included in ${}_rY_{hj}$; but in calculating Z_r no noise arising from ${}_{r-1}Y_{kli}$ must be included, for any such noise has already been included in Z_{r-1}; that is to say, in calculating the component of the form (8.6), in which (pq) is (hj), G_{pq} must exclude ${}_{r-1}G_{kli}$.

Moreover Section 10 does not specifically mention the possibility that the signal generator may be connected to the input terminals of the first stage, or that the output terminals of stage $(r-1)$ may be connected to the input terminals of stage r, not directly, but through a transformer. In practice a transformer is almost always inserted at least between the signal generator and the first stage, and performs a most important function. The noise factor depends, when all other characteristics are fixed, on the impedance of the signal generator; the effective value of this impedance can be changed by changing the transformation ratio of the transformer. A large part of the practical adjustment of the amplifier to optimum performance consists therefore in varying the transformation ratio of this transformer so as to make the noise factor a minimum for that variation.

Let us suppose, then, that a *PT* (Section 7) has its secondary terminals connected directly to the input terminals (h, j) of the first stage, and its primary terminals to those of the signal generator, the y's of the transformer being adjusted so that, when the Y_1 of Section 7 is Y_s the admittance of the generator Y_2 is Y_u. (Y_u is the quantity so denoted in Sections 4, 5.) Then the (h, j) terminals are connected effectively, by virtue of Thévenin's theorem, by an impedor of admittance Y_u in series with the E.M.F applied by the transformer. The gain from the terminals of this E.M.F. to the (k, l) terminals (the output terminals of the first stage) is therefore, by (3.9), $Y_u \cdot {}_1\rho_{kl,hj}$, where ${}_1\rho_{kl,hj}$ must be calculated with Y_u replacing Y_s. If A_1 is the gain from the terminals of the signal generator to the (k, l) terminals, we have, from (7.7)

$$|A_1|^2 = \frac{|Y_s|^2}{|Y_u|^2}\cdot\frac{G_u}{G_s}\cdot|Y_u|^2|{}_1\rho_{kl,hj}|^2 \quad (11.1)$$

Therefore, from (6.1)

$$R_s|A_1|^2 = G_u|{}_1\rho_{kl,hj}|^2 \quad .. \quad .. \quad (11.2)$$

Accordingly the effect of the *PT* is simply to substitute quantities with suffix u for the corresponding quantities with suffix s in the integrand of (10.3) or (10.5). It should be noted that the quantities in (11.2) are functions of frequency.

From (11.2) it is obvious that, if the signal generator provides a sinusoidal E.M.F. of frequency f_0, and A_s is the gain for this signal from the terminals of the generator to the (k, l) terminals—which is what is usually called the signal gain—then the foregoing argument gives

$$|A_s|^2 = \frac{G_u(f_0)}{R_s(f_0)}|{}_1\rho_{kl,hj}(f_0)|^2 \quad .. \quad (11.3)$$

where, of course, (f_0) after a quantity indicates that its value is to be taken at the frequency f_0.

Since a perfect transformer cannot be exactly realised, we must, in accordance with Section 7, introduce a conductor of conductance γ in parallel with Y_u. γ will vary when Y_u is varied by adjustment of the transformer, and, when the adjustment is made,

will in general vary with frequency, like all the other admittances. In making the Thévenin transformation γ may be excluded from the PT in which case it does not appear in Y_u but its presence will have to be taken into account in estimating the ρ's; further by (8.1), it will provide a component of the noise $4k'T\int_0^\infty \gamma|\rho_{kl,hj}|^2\,df$. It is relevant here to observe that in most valves operating at high frequencies there is appreciable leakage between the (h, j) terminals, which may be represented by a conductor (whose conductance may vary with frequency) between these terminals. So long as there is no inductance in the leads to the electrodes, this conductor is in parallel with that which represents the imperfection of the input transformer; the effect of both may be represented by a single γ.

If there is a transformer between the $(r-1)$ th and rth stages, it may be regarded as part of either stage, or even divided into two parts, one belonging to one stage and the other to the other, so long as each stage still has definite input and output terminals. Different procedures may be suitable in different circumstances. If it is regarded as part of the $(r-1)$th stage, it will have three effects. First, it will multiply by the same factor all the gains of the $(r-1)$th stage; second, it will change the admittance looking into the output terminals of that stage; third (since the transformer cannot be PT), it will introduce a conductance γ in parallel with the output terminals. It will almost always be convenient to regard this conductance as part of the rth stage rather than of the $(r-1)$th, that is to say as part of ${}_rY_{hj}$ rather than of ${}_{r-1}Y_{kl}$; for, as has been said, ${}_rY_{hj}$ will usually already have a part γ, whose value is merely modified by a part arising from the imperfection of the transformer. We have therefore to consider only the first two effects. Let dashed quantities refer to the stage without the transformer, undashed quantities to the stage with the transformer. Then the second effect will be to substitute ${}_{r-1}Y_{kli}$ for ${}_{r-1}Y'_{kli}$, the relation between these two quantities being determined by the y's of the PT. By (7.7) we shall then have, for the first effect,

$$\frac{|A_{kl,pq}|^2}{|A'_{kl,pq}|^2}=\frac{|\rho_{kl,pq}|^2}{|\rho'_{kl,pq}|^2}=\frac{|{}_{r-1}Y'_{kli}|^2}{|{}_{r-1}Y_{kli}|^2}\cdot\frac{{}_{r-1}G_{kli}}{{}_{r-1}G'_{kli}}$$
$$=\frac{{}_{r-1}R_{kli}}{{}_{r-1}R'_{kli}} \qquad .. \quad .. \quad (11.4)$$

12. Noise Factor of a Common-Cathode Pentode

It will be well before we proceed to illustrate one of the apparently complicated conclusions presented in Sections 10, 11 by a concrete example. We shall take that of a single stage consisting of a common-cathode pentode without inductance in the electrode leads, fed by a signal generator through a transformer.

In the notation of Section 5, the (k, l) terminals are (1, 3), the (h, j) terminals are (1, 2). The sources of noise are the impedors Y_u and γ between (1, 2), and the electron stream and the anode load Y_v between (1, 3). We have therefore from (10.5) and (11.2)

$$N=1+\frac{\int_0^\infty \{\gamma|\rho_{13,12}|^2+(G_v+G_{13\sigma})|\rho_{13,13}|^2\}\,df}{\int_0^\infty G_u|\rho_{13,12}|^2\,df} \qquad .. \quad .. \quad (12.1)$$

where $G_{13\sigma}$ from (8.11), since $\rho_{kl,pr}$ is here zero, is given by

$$G_{13\sigma}=a\,I_0(\alpha_3{}^2\,\Gamma^2+\alpha_3\alpha_4) \qquad .. \quad (12.2)$$

Since (3.11) is true, we can omit the common denominator Δ of the ρ's, and replace them by δ's. Then, from (5.4, 6), remembering that we must now add γ to Y_{02} in (5.1), and that

$$\delta_{13,12}=\Delta_{32}\,;\quad \delta_{13,13}=\Delta_{33} \qquad .. \quad (12.3)$$

we have

$$N=1+\frac{\int_0^\infty \gamma(\alpha_3{}^2g_2{}^2+\omega^2C^2_{23})\,df+\int_0^\infty (G_v+G_{13\sigma})\{(G_u+\gamma)^2+(B_u+\omega C_2)^2\}\,df}{\int_0^\infty G_u(\alpha_3{}^2g_2{}^2+\omega^2C^2_{23})\,df} \qquad .. \quad (12.4)$$

where

$$C_2=C_{02}+C_{12}+C_{23}+C_{24}+C_{25} \qquad .. \quad .. \quad (12.5)$$

The problem of adjusting the input transformer, and therefore G_u,B_u, so that N is a minimum is in general very complicated. For G_u,B_u are functions of f whose form can be

varied by varying the y's of the PT; moreover γ, which is also a function of f, will vary with G_u, B_u. If, however, we are interested only in a frequency band which is so narrow that throughout all quantities may be considered constant, then (12.4) becomes

$$N = 1 + \frac{\gamma}{G_u} + \frac{(G_v + G_{13\sigma})\{(G_u + \gamma)^2 + (B_u + \omega_0 C_2)^2\}}{G_u(\alpha_3^2 g_2^2 + \omega_0^2 C^2_{23})} \quad .. \quad .. \quad (12.6)$$

where G_u, B_u are variable quantities and ω_0 refers to the centre of the band. Further, if it may be assumed that γ varies slowly with G_u, B_u, so that in adjusting them for minimum N, γ may be taken as a known constant, then the conditions for minimum N are

$$\frac{\partial N}{\partial G_u} = 0 \; ; \quad \frac{\partial N}{\partial B_u} = 0 \quad .. \quad .. \quad (12.7)$$

The second of these gives

$$B_u = -\omega_0 C_2 \quad .. \quad .. \quad .. \quad (12.8)$$

The first gives

$$G_u^2 = \gamma G_c + \gamma^2 \quad .. \quad .. \quad (12.9)$$

where

$$G_c = \frac{\alpha_3^2 g_2^2 + \omega_0^2 C^2_{23}}{G_v + G_{13\sigma}} \quad .. \quad .. \quad (12.10)$$

Consequently

$$N_{\min} = 1 + 2\frac{\gamma}{G_c} + 2\sqrt{\frac{\gamma}{G_c}\left(1 + \frac{\gamma}{G_c}\right)} \quad .. \quad .. \quad (12.11)$$

This result shows how important is γ. For, if it were zero, the optimum value of G_u would be zero, and $N_{\min}$ would be 1. At the same time, by (11.3), the signal gain would be zero. The problem would become entirely insignificant. That there is a real optimum adjustment of the transformer in this simplified case depends wholly on the presence of a non-zero γ.

13. Effect on the Noise Factor of a Prefixed Stage

We can now approach the problem set forth in Section 1. Suppose again that the given amplifier has n stages. Then its noise factor N_D, measured with a signal generator of internal impedance Z_s, is given by (10.5). Let a pre-amplifier, which we shall call stage 0, be prefixed, and its noise factor N be measured with the same signal generator. Then, if the gains A_1 to A_n are unchanged, we have

$$N = 1 + \frac{\left(\int_0^\infty Z_0|A_1|^2 \ldots |A_n|^2\,df + \int_0^\infty Z_1|A_2|^2 \ldots |A_n|^2 df + \ldots + \int_0^\infty Z_n\,df\right)}{\int_0^\infty R_s|A_0|^2|A_1|^2 \ldots |A_n|^2\,df} \quad .. \quad .. \quad (13.1)$$

If R_s is independent of frequency, so that it can be taken outside the integral in (10.5), and if the variation of A_0 over the band is sufficiently small, then

$$N = 1 + \frac{\int_0^\infty Z_0|A_1|^2 \ldots |A_n|^2\,df}{R_s|A_0|^2\int_0^\infty |A_1|^2 \ldots |A_n|^2\,df} + \frac{\int_0^\infty Z_1|A_2|^2 \ldots |A_n|^2\,df + \ldots + \int_0^\infty Z_n\,df}{R_s|A_0|^2\int_0^\infty |A_1|^2|A_2|^2 \ldots |A_n|^2\,df} \quad .. \quad .. \quad (13.2)$$

$$= 1 + \frac{\int_0^\infty Z_0|A_D|^2 \cdot df}{R_s|A_0|^2\int_0^\infty |A_D|^2\,df} + \frac{N_D - 1}{|A_0|^2} \quad .. \quad .. \quad (13.3)$$

where $A_D = A_1 \cdot A_2 \ldots A_n$ .. (13.4)

is the whole gain of the amplifier from the terminals of the signal generator feeding stage 1 to the (k, l) terminals of stage n. Further in virtue of the conditions of the problem, A_D is likely to be appreciably constant through the pass-band, although the components A_r may vary. Accordingly we may omit the integral signs in (13.3), and arrive at the simple formula

$$N = 1 + \frac{Z_0}{R_s|A_0|^2} + \frac{N_D - 1}{|A_0|^2} \quad .. \quad (13.5)$$

It should be noted that (13.5) has been derived on the assumption that the A_r's are unchanged by the prefixing of stage 0. This would imply that the output admittance of stage 0 is equal to the admittance of the generator and is generally not true.

It is most improbable that the gains A_1 to A_n will be unchanged between the measurement of N_D and the measurement of N. The

conditions when N_D is measured are likely to be those shown to the right of the dotted line in Fig. 7. The signal generator is applied to the (h, j) terminals of stage 1 through a transformer T_1, and the y's of this transformer are adjusted to make N_D a minimum for variation of those y's. If, when N is measured, the transformer T_1 is removed

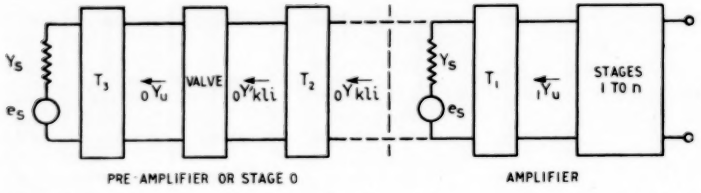


Fig. 7. *The use of a pre-amplifier modifies the conditions for the following stage.*

with the signal generator, A_1 is reduced by the gain in the transformer. If T_1 is left in place, and the (k, l) terminals of stage 0 are connected to its input in place of those of the signal generator, the input admittance of T_1 is changed from Y_s to ${}_0Y_{kl}$, which is not in general even nearly the same as Y_s. Accordingly the gain of T_1 will change, and so will the admittance of ${}_1Y_u$, which is part of the admittance ${}_1Y_{hj}$; accordingly A_1 will change. Further ${}_1Y_{kl}$ will change, and therefore A_2 will change; and so on; and the amplifier will no longer be adjusted for minimum N_D. (It is not certain that adjustment for minimum N will involve adjustment for minimum N_D; but that represents at least a possibility to be explored.)

In order to avoid this difficulty, a transformer T_2 may be inserted between the output of stage 0 and the input of stage 1 when the signal generator is removed, as shown to the left of the dotted line. (Actually, of course, if this is done, T_1, T_2 are likely to be combined.) T_2 transforms the admittance ${}_0Y'_{kli}$, looking into the (k, l) terminals of stage 0 in the absence of the transformer, into ${}_0Y_{kli}$, the admittance looking into the output terminals of the transformer, which are now the (k, l) terminals of stage 0. If it could be arranged that ${}_0Y_{kli} = Y_s$, N_D would again be adjusted to the minimum and A_1 to A_n would retain their original values. But in general this cannot be arranged; ${}_0Y_{kli}$ can be made equal to Y_s at one frequency, but not at all frequencies.

However, if the conditions for (10.8), (13.3, 5) prevail, and the gain of the prefixed valve may be considered constant over the relevant range, it is likely that all the characteristics of stage 0, and in particular, ${}_0Y'_{kli}$, can be considered similarly constant. Then, so long as Z_s is a pure resistance, the desired end can be obtained; and, so long as the condition for minimum N involves the condition that N_D shall be a minimum, minimum N can be obtained by varying only the transformer T_2 between the signal generator and the (h, j) terminals of stage 0.

By (7.7), if A_0 is the gain of stage 0 without T_2 and A_0 with it,

$$\frac{|A_0|^2}{|A'_0|^2} = \frac{R_s}{{}_0R'_{kli}} \quad .. \quad .. \quad .. \quad (13.6)$$

A'_0 is the A_s of (11.3). Using (6.1b) and (3.16), we have

$$|A_0|^2 = |A'_0|^2 \cdot R_s \cdot \frac{|{}_0Y'_{kli}|^2}{{}_0G'_{kli}} = \frac{R_s|A'_0|^2}{{}_0G'_{kli}|{}_0\rho'_{kl,kl}|^2} \quad .. \quad .. \quad (13.7)$$

When T_2 is introduced, all the gains involved in Z_0 will change in the ratio A_0/A'_0; for they are all gains to the (k, l) terminals of stage 0. Hence, in the second term on the right of (13.3, 5), we may write A'_0 in the denominator so long as we use dashed quantities, when we expand Z_0. Hence (13.5) becomes

$$N = 1 + \frac{Z_0}{R_s|A'_0|^2} + \frac{(N_D - 1){}_0G'_{kli}|{}_0\rho'_{kl,kl}|^2}{R_s|A'_0|^2} \quad .. \quad .. \quad (13.8)$$

If we now expand Z_0 by (8.7), and, for symmetry, substitute for $R_s|A'_0|^2$ from (11.3), we have

$$N = 1 + \frac{\Sigma_{pq}(G_{pq} + G_{pq\sigma})|{}_0\rho'_{kl,pq}|^2 + (N_D - 1){}_0G'_{kli}|{}_0\rho'_{kl,kl}|^2}{G_u|{}_0\rho'_{kl,hj}|^2} \quad .. \quad .. \quad (13.9)$$

Now the Σ_{pq} contains a term $G_{kl}|\rho'_{kl,hj}|^2$ where G_{kl} is the conductance of the load which in Sections 4, 5 is called G_v. Hence, in this particular case, the effect on N of the amplifier following the pre-amplifier is the same as if the amplifier produced no noise, but added to the conductance of the load of the pre-amplifier a conductance $(N_D - 1){}_0G'_{kli}$, where ${}_0G'_{kli}$ is the conductance looking into the pre-amplifier from the input terminals of the transformer T_2.

14. Conclusion

In Section 13, some attempt has been made to simplify the fundamental formula (10.5) for particular cases. The validity of the assumptions we make for this purpose are difficult to assess, although it is quite usual for such assumptions to be made.

It is hoped that the theory developed here will allow the validity or otherwise of these assumptions to be verified.

It is hoped also that the theory will make possible the inclusion of all circuit elements (such as stray lead inductances) in the analysis with a view to assessing their effect.

The application of the theory with these objects in mind leads to rather lengthy algebra, and it is proposed to deal with some specific cases in later papers.

15. Acknowledgement

The Authors desire to tender their acknowledgement to the General Electric Company and the Marconiphone Company, on whose behalf the work which led to this publication was done.

CORRECTION

Equation (2.3) of Section 2 should read:—

$$\overline{(v-\bar{v})^2} = \frac{2k'T}{\pi}\int_0^\infty R_{pq}|A_{kl,\,pq}(j\omega)|^2 d\omega$$

REFERENCES

[1] N. R. Campbell and V. J. Francis. "A Theory of Valve and Circuit Noise." *J.I.E.E.*, Vol. 93, Part III, No. 21, Jan. 1946, p. 45.

[2] N. R. Campbell, V. J. Francis and E. G. James. "Linear Single-Stage Valve Circuits," *Wireless Engineer*, July 1945, Vol. XXII, p. 333.

[3] D. M. O. North. "Fluctuations in Space-Charge-Limited Currents at Moderately High Frequencies." *R. C. A. Review*, April 1940, Vol. IV. p. 441.

WIRELESS PATENTS

A Summary of Recently Accepted Specifications

The following abstracts are prepared, with the permission of the Controller of H.M. Stationery Office, from Specifications obtainable at the Patent Office, 25, Southampton Buildings, London, W.C.2, price 1/- each.

ACOUSTICS AND AUDIO-FREQUENCY CIRCUITS AND APPARATUS

573 420.—Capacitance microphone in which one of the plates is relatively massive, for the detection and measurement of mechanical vibrations.

Standard Telephones and Cables Ltd., V. J. Terry, and E. A. Foulkes. Application date 2nd November, 1943.

573 687.—Flexible acoustic fitting for coupling a piezo-electric crystal to a sound-diaphragm.

Brush Crystal Co. Ltd. (assignees of A. L. W. Williams and D. Domizi). Convention date (U.S.A.), 20th March, 1939.

573 688.—Piezo-electric or other drive for a small diaphragm, particularly suitable for a deaf-aid appliance.

Brush Crystal Co. Ltd. (assignees of A. L. W. Williams and D. Domizi). Convention date (U.S.A.) 20th March, 1939.

DIRECTIONAL WIRELESS

573 020.—Variable-frequency type of radio-altimeter arranged to transmit a beam making a conical "sweep" so as also to indicate the speed and course of the aircraft.

R. C. A. Photophone Ltd. and H. W. H. Hodgkins. Application date 3rd July, 1942.

573 436.—Construction and arrangement of the screening-elements of a directional aerial of the kind in which the radiating sections of a conductor are separated by non-radiating sections.

The General Electric Co. Ltd., and R. J. Clayton. Application date 19th August, 1942.

573 744.—Headphone-receiver for navigational signals of the A—N type, with means for modifying the different signal-amplitudes, in order to ease the strain on the pilot's hearing as he approaches the beacon.

Standard Telephones and Cables Ltd. and L. J. Heaton-Armstrong. Application date 24th November, 1943.

TELEVISION CIRCUITS AND APPARATUS

FOR TRANSMISSION AND RECEPTION

572 338.—Louvred device forming a viewing screen for a cathode-ray television receiver.

Philco Radio and Television Corpn. (assignees of G. Zindell, Jr.). Convention date (U.S.A.). 10th September, 1942.

573 008.—Television system in which the image is produced in three dimensions, and is so projected in space (and not upon a flat screen).

J. L. Baird. Application dates 26th August, 27th September, 7th and 11th October, and 26th November, 1943 and 9th February, 1944.

573 226.—Multiple-gun cathode ray tube used for scanning a large-sized television screen in successive sections.

A. Soloman. Application date 18th October, 1943.

SIGNALLING SYSTEMS OF DISTINCTIVE TYPE

572 918.—Multi-channel signalling system in which all the frequencies required for inter-communication are controlled by the selected harmonics of a single crystal-oscillator.

The General Electric Co. Ltd., N. R. Bligh, D. M. Heller and L. C. Stenning. Application dates 27th April and 1st July, 1942.

CONSTRUCTION OF ELECTRONIC-DISCHARGE DEVICES

572 540.—Construction and arrangement of the focusing-electrodes for a discharge tube of the velocity-modulation type.

C. S. Bull. Application date 29th April, 1940.

572 813.—Spacing and aligning the focusing and other electrodes of a cathode-ray tube by means of protuberances formed inside and outside the glass bulb.

Ferranti Ltd., S. Jackson, A. L. Chilcot, M. E. Sions and R. W. Sutton. Application date 29th May, 1943.

573 032.—Construction and mounting of the magnetic deflecting coils of a cathode-ray tube.

The British Thomson-Houston Co. Ltd. (communicated by the General Electric Co.). Application date 24th September, 1943.

573 097.—Electron-discharge tube embodying a "Hertzian gap" for detecting and amplifying, or generating, ultra-short waves.

H. A. Malpas. Application date 9th November, 1943.

573 170.—Construction of high-power valve of the kind in which the filament-heating leads consist of two coaxial copper tubes.

Standard Telephones and Cables, Ltd. and W. T. Gibson. Application date 31st March, 1944.

573 193.—Construction of a hollow resonator tube for generating micro-waves by the phase-control of an electron-stream.

Patelhold Patentverwertungs and Elektroholding A.G. Convention date (Switzerland), 1st September, 1942.

573 205.—Arrangement of the control and deflecting electrodes of an electron-discharge device of the beam type, suitable for frequency-doubling or as a relay.

J. H. O. Harries. Application date 5th September, 1940.

573 206.—Construction and mounting of an interwoven spiral cathode, suitable for valves of the beam or deflection type.

J. H. O. Harries. Application date 5th September, 1940.

SUBSIDIARY APPARATUS AND MATERIALS

572 491.—Process for laying an insulating film on sheets of ferrous magnetic material intended for making laminated transformer-cores.

G. R. Shepherd (communicated by Westinghouse Electric International Co.). Application date 25th January, 1944.

572 658.—Process for removing occluded air or gas in the course of the manufacture of dry cells.

Siemens Bros. & Co. Ltd., T. H. Larke and F. G. Kny-Jones. Application date 15th April, 1944.

572 739.—The use of sliding dielectric blocks, having a selected permittivity, as impedance-transformers along a transmission line.

The General Electric Co. Ltd. and D. C. Espley. Application date 11th November, 1941.

572 846.—Selective filter-network having six terminals and comprising both high-pass and low-pass sections.

The General Electric Co. Ltd. and R. O. Rowlands. Application date 19th July, 1943.

572 848.—Assembling and bonding multiple dry-plate rectifier units.

The General Electric Co. Ltd., H. C. Turner and J. Chamberlain. Application date 21st July, 1943.

572 870.—Construction of right-angled branches or stubs for a transmission-line of the coaxial type.

Bruno Patents Inc. Convention dates (U.S.A.) 27th August and 3rd December, 1942.

572 881.—A wave-guide in which part of the guiding surface comprises a quarter-wave "open" grid, which prevents the escape of energy whilst allowing power measurements to be made.

J. Collard. Application date 14th September, 1943.

572 884.—Closed chain of hard-valves operating as a "counter" circuit, and indicating, on the scale of ten, events of the order of 100 000 per second.

Cinema-Television, Ltd. and T. C. Nuttall. Application date 15th February, 1941.

572 999.—Selenium rectifier in which the sensitive material is applied to the electrode from a colloidal suspension in a volatile carrier, the latter then being evaporated.

Standard Telephones and Cables, Ltd. (assignees of O. Saslaw). Convention date (U.S.A.) 24th April, 1942.

573 030.—Resilient metal-ring so shaped as to minimise the tendency of vibration to loosen a thermionic valve in its holder.

A. C. Cossor, Ltd. and A. Levin. Application date 17th September, 1943.

573 099.—Two-core R.F. transformer having a wide band-pass characteristic, and designed to facilitate precision production in quantity.

Philco Radio and Television Corporation (assignees of C. T. McCoy). Convention date (U.S.A.) 29th June, 1943.

573 108.—Electric soldering-iron in which the heat-storing element is under thermostatic control.

Standard Telephones and Cables, Ltd. (trading as Stanelco Products) and J. Handley. Application date 29th November, 1943.

573 124.—Impedance-matching transformer comprising a transmission-line element terminated by a non-reactive load, and a second transmission-line element adjustably shunted across the first.

Standard Telephones and Cables, Ltd. and D. C. Rogers. Application date 1st December, 1943.